1 Background

Two-dimensional (2D) turbulence has been of considerable interest because it is believed to be an apt paradigm for geophysical flows.

Moreover, vortex motions are fundamental elements of the 2D turbulence in the physical space, because:

- 2D turbulence is full of vortices,
- such vortices mutually advect,
- two like-signed vortices merge into a single vortex,
- and a single elliptic vortex approaches circular one.

In this study, we reconsider a research on the axisymmetrization process of the elliptic vortex of the incompressible 2D fluid by Melander et al. (1987), numerically.

Thus the governing equation is the following vorticity equation:

$$\frac{\partial \omega}{\partial t} + \frac{\partial \omega}{\partial x} \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} = \nu \nabla^2 \omega$$

where $$\omega = \nabla \psi$$.

2 Axisymmetrization of the elliptic vortex

In this section, we briefly review the kinematics of the axisymmetrization of the elliptic vortex proposed by Melander et al. (1987).

- An elliptic vortex evolves to an axisymmetric vortex.
- The vortex ejects filaments.

2.1 The relationship between the axisymmetrization and $$\phi_d$$

Melander et al. (1987) introduced the difference angle $$\phi_d$$ between the orientation of the ellipse with a vorticity contour and that with a streamline contour as a diagnostic tool for understanding the axisymmetrization of an elliptic vortex:

- solid line: vorticity contour
- broken line: streamline contour
- $$\phi_d = \phi_v - \phi_s$$
- $$\phi_v$$: orientation of the ellipse with vorticity contour
- $$\phi_s$$: orientation of the ellipse with streamline contour

In the earlier time of evolution, the division between the vortex core and the filaments is obscure from the morphology of the vorticity field. However, in the later time evolution, $$\phi_d$$ seems to be a reasonable estimate of the boundary of the core and the filaments.

- The contribution of the filaments is larger than that of the core.
- The contribution of the filaments at 2.5 ≤ $$t$$ ≤ 5.0.

3 Numerical estimate of $$\phi_d$$

3.1 Procedure

We investigate the contribution of the filaments to the $$\phi_d$$ with the following procedure:

1. Divide the vorticity field into the vortex core and the filaments.
2. Here, the filaments are determined by the morphology of the vorticity field.
3. Calculate the stream function induced by the vortex core.
4. Fit ellipse to the vorticity contour and the streamline, then calculate the difference angle of the principal axes of the ellipses.

- We use the fitting method proposed by Fitzgibbon et al. (1999).

3.2 Settings for the simulation

- initial aspect ratio of the vortex:
- number of grid points: 3072
- viscous coefficient: $$\nu = 5 \times 10^{-5}$$
- maximum of the initial vorticity $$\omega_0 = 10$$
- initial aspect ratio of the vortex: $$\sqrt{0.7}$$
- the other initial conditions: same as Melander et al. (1987) and the compact support case in Kinnum and Hirning (2001).

3.3 Determination of the core-filaments boundary

Red solid lines are the boundaries between the vortex core and filaments.

3.4 Results

3.4.1 Contribution of the vortex core

A natural question is raised: How large do the filaments contribute to $$\phi_d$$?

$$\Rightarrow$$ We examine this point numerically.

3.4.2 Contribution of the filaments

Fig.1 and Fig.2 shows that:

- amplitude of $$\phi_d$$ is almost constant during the evolution.
- $$\phi_d$$ is positive in the early time of evolution, and rapidly oscillates compared to $$\phi_v - \phi_s$$.
- The contribution of the filaments largely depends on the definition of the filament in 0 ≤ $$t$$ ≤ 1.0.

- We choose $$r ≈ 0.7$$ as the boundary between the core and the filaments. In the early time even the division between the vortex core and the filaments is obscure from the morphology of the vorticity field. However, in the later time evolution, $$r ≈ 0.7$$ seems to be a reasonable estimate of the boundary of the core and the filaments.

- The contribution of the filaments is larger than that of the core.
- The contribution of the filaments at 2.5 ≤ $$t$$ ≤ 5.0.

4 Conclusion

The contribution of the filaments to the axisymmetrization process of the elliptic vortex has been investigated quantitatively.

- We calculated the difference angle $$\phi_d$$ between orientations of elliptic vorticity and streamline contours which is a diagnostic tool for understanding the axisymmetrization of the elliptic vortex proposed by Melander, et al. (1987).

- We calculate the contribution of the vortex core and the filaments to $$\phi_d$$, the difference angle between the principal axes of the ellipses with a vorticity contour and a streamline, respectively, from the filaments and the core, respectively.

- The filaments largely contributes to $$\phi_d$$ in early stage.

- In the early time of evolution, the contributions of the filaments to $$\phi_d$$ is comparable to that of the vortex core.

- In the later time, the contribution of the filaments seems to be less important.

We confirm numerically that the filaments play an important role for the axisymmetrization of the elliptic vortex.

References