ELECTRICAL AND THERMAL CONDUCTIVITIES OF DENSE MATTER IN THE CRYSTALLINE LATTICE PHASE. III. INCLUSION OF LOWER DENSITIES

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ABSTRACT

The electrical and thermal conductivities of dense matter in the crystalline lattice phase due to phonon scattering are calculated including the lower densities $10^0 - 10^4$ g cm^{-3}. By this inclusion the data of the electrical and thermal conductivities of dense matter in the crystalline lattice phase will be complete for the density range $10^{0.0} - 10^{12.7}$ g cm^{-3}. Analytic fitting formulae are given to facilitate applications of the present numerical results.

Subject headings : atomic processes — dense matter — stars : neutron —

stars :white dwarfs

1. INTRODUCTION

Nearly a decade has passed since the paper of Itoh et al. (1984a) was published. This paper together with the papers of Itoh et al. (1983) and Mitake, Ichimaru, & Itoh (1984) has given significantly improved results of the conductivities upon some of the previous results reported in the papers of Flowers & Itoh (1976), Flowers & Itoh (1981), Yakovlev & Urpin (1981), and Raikh & Yakovlev (1982). Recently Itoh & Kohyama (1993) have given results on the conductivities of dense matter in the crystalline lattice phase due to impurity scattering.

In recent years white dwarf asteroseismology opened up a new fertile land of astrophysics (Bradley & Winget 1991; Bradley,Winget, & Wood 1992). The internal structure of white dwarfs is now critically tested by a wealth of asteroseismological data. Consequently, the basic physics data which goes into white dwarf models needs to be sufficiently accurate that it should live up to the standard required by the asteroseismological data.

In the paper of Itoh et al. (1984a) electrical and thermal conductivities of dense matter in the crystalline lattice phase due to phonon scattering have been calculated for the density range $10^4 - 10^{12}$ g cm^{-3} and for the neutron star matter. Accurate analytic fitting formulae have been given to facilitate the application of the numerical results. Extrapolation of the fitting formulae to the lower densities $10^0 - 10^4$ g cm^{-3}

gives moderately accurate results of the conductivities. However, of course it is best to calculate the conductivities for the densities $10^0 - 10^4$ g cm^{-3} and give an accurate analytic fitting formula. In view of the recent developments in the asteroseismology of white dwarfs, this task appears to be worth while.

The present paper is organized as follows. The calculation is presented in §2. The numerical results are given in §3. Concluding remarks are given in §4.

2. CALCULATION

In this paper we shall closely follow the method reported in the paper of Itoh et al. (1984a) and calculate the electrical and thermal conductivities of dense matter in the crystalline lattice phase due to phonon scattering for the density range $10^{0.0} - 10^{12.7}$ g cm^{-3}. The present results for the density range $10^4 - 10^{12}$ g cm^{-3} are the same as reported in the paper of Itoh et al. (1984a). The case for the neutron star matter has been already reported in the paper of Itoh et al. (1984a). Therefore, we shall not repeat the calculation for this case. We just mention an erratum to the paper of Itoh et al. (1984a) that has to be taken into account for the use of the fitting formulae of the conductivities.

We shall consider the case that the atoms are completely pressure-ionized. The corresponding condition is expressed by

$$E_{\mathrm{F}}(\rho) \geq Z^2 \ \mathrm{Ry}, \tag{1}$$

where $E_{\mathrm{F}}(\rho)$ is the electron Fermi level at zero temperature and at a given mass density ρ. Numerically, the condition (1) can be rewritten as

$$\rho \geq 0.378 A Z^2 \ \ \mathrm{g\ cm^{-3}}, \tag{2}$$

where Z and A are the atomic number and mass number of the atom considered, respectively. Thus we have $\rho \geq 6.05$ g cm^{-3} for ^{4}He, $\rho \geq 1.63 \times 10^2$ g cm^{-3} for

^{12}C, $\rho \geq 3.87 \times 10^2$ g cm^{-3} for ^{16}O, and $\rho \geq 1.43 \times 10^4$ g cm^{-3} for ^{56}Fe. One should note that these conditions must be fulfilled to guarantee the complete pressure ionization, although we present the numerical results for the whole density range $10^{0.0} - 10^{12.7}$ g cm^{-3} assuming complete pressure ionization.

We further restrict ourselves to the density-temperature region in which electrons are strongly degenerate. This condition is expressed as

$$T \ll T_{\mathrm{F}} = 5.930 \times 10^9 \left[\left[1 + 1.018 (Z/A)^{2/3} \rho_6^{2/3} \right]^{1/2} - 1 \right] \mathrm{K}, \tag{3}$$

where T_{F} is the Fermi temperature and ρ_6 is the mass density in units of 10^6 g cm^{-3}. For the ionic system, we consider the case that it is in the crystalline lattice state. The most recent criterion corresponding to this condition is given by (Ogata & Ichimaru 1987)

$$\Gamma \equiv \frac{Z^2 e^2}{a k_{\mathrm{B}} T} = 2.275 \times 10^{-1} \frac{Z^2}{T_8} \left(\frac{\rho_6}{A} \right)^{1/3} \geq 180, \tag{4}$$

where $a = [3/(4\pi n_i)]^{1/3}$ is the ion-sphere radius and T_8 is the temperature in units of 10^8 K.

The relativistic extension of the expressions for the electrical and thermal conductivities due to degenerate electrons has been given by Flowers & Itoh (1976). The electrical conductivity σ and thermal conductivity κ are related to the effective electron collision frequencies ν_σ and ν_κ by

$$\sigma = \frac{e^2 n_e}{m^* \nu_\sigma} = 1.525 \times 10^{20} \frac{Z}{A} \rho_6 \left[1 + 1.018 \left(\frac{Z}{A}\rho_6\right)^{2/3}\right]^{-1/2} \frac{10^{18}\mathrm{s}^{-1}}{\nu_\sigma} \mathrm{s}^{-1}, \tag{5}$$

$$\kappa = \frac{\pi^2 k_\mathrm{B}^2 T n_e}{3 m^* \nu_\kappa} = 4.146 \times 10^{15} \frac{Z}{A} \rho_6 \left[1 + 1.018 \left(\frac{Z}{A}\rho_6\right)^{2/3}\right]^{-1/2} T_8 \frac{10^{18}\mathrm{s}^{-1}}{\nu_\kappa} \ \mathrm{ergs\ cm^{-1}s^{-1}K^{-1}}, \tag{6}$$

where n_e is the number density of eletrons and m^* is the relativistic effective mass of an electron at the Fermi surface. In this paper we are interested in the scattering of electrons by phonons. The collision frequencies ν_σ and ν_κ due to one-phonon processes can be calculated by the variational method (Flowers & Itoh 1976; Yakovlev & Urpin 1981; Raikh & Yakovlev 1982) as

$$\nu_{\sigma,\kappa} = \frac{e^2}{\hbar v_\mathrm{F}} \frac{k_\mathrm{B} T}{\hbar} F_{\sigma,\kappa} = 9.554 \times 10^{16} T_8 \left\{1 + \frac{1}{1.018\left[(Z/A)\rho_6\right]^{2/3}}\right\}^{1/2} F_{\sigma,\kappa} \ \mathrm{s}^{-1}, \tag{7}$$

$$F_{\sigma,\kappa} = \frac{2\gamma^2}{S^2} \int \frac{dS dS'}{k^4 \mid \varepsilon(k,0) \mid^2} \left[1 - \left(\frac{\beta k}{2k_\mathrm{F}}\right)^2\right] e^{-2W(k)} \mid f(k) \mid^2 \sum_{s=1}^{3} [\mathbf{k} \cdot \hat{\epsilon}_s(\mathbf{p})]^2 (e^{z_s} - 1)^{-2} e^{z_s} g_{\sigma,\kappa} \ . \tag{8}$$

In the above the integral is over the areas of the Fermi surface, $\mathbf{k}$ is the momentum transfer, $\hat{\epsilon}_s(\mathbf{p})$ is the polarization unit vector of a phonon with momentum $\mathbf{p}$ and polarization s, and

$$\gamma \equiv \frac{\hbar \omega_p}{k_\mathrm{B} T} = 7.832 \times 10^{-2} \frac{Z}{A} \frac{\rho_6^{1/2}}{T_8} = 0.3443 \frac{\rho_6^{1/6}}{A^{2/3} Z} \Gamma, \tag{9}$$

$$\beta \equiv \frac{\hbar k_{\rm F} c}{E_{\rm F}} = \left\{ 1 + \frac{1}{1.018[(Z/A)\rho_6]^{2/3}} \right\}^{-1/2}, \tag{10}$$

$$z_s \equiv \frac{\hbar \omega_s(\mathbf{p})}{k_{\rm B} T}, \tag{11}$$

$$g_\sigma = k^2, \tag{12}$$

$$g_\kappa = k^2 - \frac{k^2 z_s^2}{2\pi^2} + \frac{3k_{\rm F}^2 z_s^2}{\pi^2}, \tag{13}$$

ω_p being the ionic plasma frequency. The momentum conservation requires $\mathbf{k} = \pm\mathbf{p} +$ K, where $\mathbf{K}$ is the reciprocal-lattice vector for the Brillouin zone to which $\mathbf{k}$ is confined.

In equation (8) we have included the dielectric screening function due to relativistically degenerate electrons $\varepsilon(k,0)$, the Debye - Waller factor $e^{-2W(k)}$, and the atomic form factor $f(k)$. Yakovlev & Urpin (1981) and Raikh & Yakovlev (1982) have used the Thomas - Fermi screening and set $e^{-2W(k)} = 1,\ f(k) = 1$.

The static longitudinal dielectric function due to relativistically degenarate electrons (Jancovici 1962) is given by

$$\varepsilon(q,0) = 1 + \left(\frac{2}{3\pi^2}\right)^{2/3} \frac{r_s}{q^2} \left[\frac{2}{3}(1+x^2)^{1/2} - \frac{2q^2x}{3}\sinh^{-1}x + (1+x^2)^{1/2}\frac{x^2+1-3q^2x^2}{6qx^2}\ln\left|\frac{1+q}{1-q}\right|\right.$$

$$\left. + \frac{2q^2x^2-1}{6qx^2}(1+q^2x^2)^{1/2}\ln\left|\frac{q(1+x^2)^{1/2}+(1+q^2x^2)^{1/2}}{q(1+x^2)^{1/2}-(1+q^2x^2)^{1/2}}\right|\right], \tag{14}$$

$$q = \frac{k}{2k_{\rm F}}, \tag{15}$$

$$x = \frac{\hbar k_{\rm F}}{m_e c} = \frac{1}{137.0}\left(\frac{9\pi}{4}\right)^{1/3} r_s^{-1}, \tag{16}$$

$$r_s = 1.388 \times 10^{-2}\left(\frac{A}{Z\rho_6}\right)^{1/3}. \tag{17}$$

The electron Fermi wavenumber is expressed as

$$k_{\rm F} = 2.613 \times 10^{10}\left(\frac{Z}{A}\rho_6\right)^{1/3} \ {\rm cm}^{-1}. \tag{18}$$

The Debye - Waller factor is written as

$$2W(k) = \sum_{k's'} \frac{\hbar[\mathbf{k}\cdot\hat{\epsilon}_{s'}(\mathbf{k}')]^2}{2MN\omega_{s'}(\mathbf{k}')}\coth\left(\frac{\hbar\omega_{s'}(\mathbf{k}')}{2k_{\rm B}T}\right), \tag{19}$$

where M is the mass of an ion and N is the number of ions in the lattice. Itoh et al. (1984b) have obtained the practical formula for the Debye - Waller factor for the pure Coulomb bcc lattice which satisfies the frequency moment sum rules for both $k_{\rm B}T \gg \hbar\omega_p$ and $k_{\rm B}T \ll \hbar\omega_p$ (Coldwell-Horsfall & Maradudin 1960; Pollock & Hansen 1973). The practical formula for the Debye - Waller factor reads as

$$2W(k) = 241.5q^2\alpha\gamma^{-3}\int_0^{0.4306\gamma}\left(\frac{1}{e^z-1}+\frac{1}{2}\right)zdz, \tag{20}$$

$$\alpha \equiv \frac{\hbar^2 k_{\rm F}^2}{2Mk_{\rm B}T} = 1.656\times 10^{-2}\frac{1}{AT_8}\left(\frac{Z}{A}\rho_6\right)^{2/3} = 0.2114\frac{\rho_6^{1/6}}{A^{2/3}Z^{1/3}}\gamma. \tag{21}$$

The practical approximations of the Debye function that appears in equation (20) are written as follows:

$$\int_0^x\left(\frac{2}{e^z-1}+\frac{1}{2}\right)zdz \approx x\left(1+\frac{1}{6\cdot 3!}x^2-\frac{1}{30\cdot 5!}x^4+\frac{1}{42\cdot 7!}x^6-\frac{1}{30\cdot 9!}x^8+\frac{5}{66\cdot 11!}x^{10}\right)$$

$$\text{for } x < 2.5, \tag{22}$$

$$\int_0^x\left(\frac{1}{e^z-1}+\frac{1}{2}\right)zdz \approx \frac{\pi^2}{6}+\frac{1}{4}x^2-\sum_{n=1}^{5}\frac{1}{n}\left(x+\frac{1}{n}\right)e^{-nx} \quad \text{for } x > 2.5. \tag{23}$$

The errors of formulae (22) and (23) at $x = 2.5$ are both smaller than 10^{-5} .

When the de Broglie wavelength of the electron becomes short enough to be comparable to the nuclear radius, we have to take into account the finite nuclear size corrections for the Coulomb scattering. This is done by multiplying the matrix element by the atomic form factor

$$f(q) = -3\frac{(2k_{\rm F}r_cq)\cos(2k_{\rm F}r_cq)-\sin(2k_{\rm F}r_cq)}{(2k_{\rm F}r_cq)^3}. \tag{24}$$

The charge radius of the nucleus is represented by

$$r_c = 1.15 \times 10^{-13} A^{1/3} \text{ cm.} \tag{25}$$

3. NUMERICAL RESULTS

The phonon spectra are modified by the screening due to electrons. The longitudinal optical phonon turns into an acoustic phonon in the long-wavelength limit, whereas the original transverse acoustic phonons are little affected by the electron screening (Pollock & Hansen 1973). Because the low-frequency transverse phonons play dominant roles in the resistivity of dense stellar matter, we neglect the effects of the electron screening on the phonon spectra and use the frequency moment sum rules for the pure Coulomb lattice.

As we consider the case in which the Fermi sphere is much larger than the Debye sphere, $(k_{\rm F}/k_{\rm D})^3 = Z/2 \gg 1$, *Umklapp* processes contribute to the scattering dominantly, and the vector **k** in equation (8) most probably falls in a Brillouin zone distant from the first zone. When we perform an integration within a single distant zone corresponding to the reciprocal-lattice vector **K**, we can make an approximation $\mathbf{k} = \mathbf{K}$ in the integrand and carry out an integration over **p** within the first zone only.

Here we follow the semianalytical approach adopted by Yakovlev & Urpin (1981) and also by Raikh & Yakovlev (1982). We write

$$\sum_{s=1}^{3}[\mathbf{k}\cdot\hat{\epsilon}_s(\mathbf{p})]^2 z_s^n (e^{z_s}-1)^{-2} e^{z_s} \approx \frac{\pi^n k^2}{\gamma^2} G^{(n)}(\gamma), \tag{26}$$

$$G^{(n)}(\gamma) = \frac{\gamma^2}{3V_{\rm B}\pi^n}\sum_{s=1}^{3}\int d\mathbf{p}\ z_s^n(e^{z_s}-1)^{-2}e^{z_s}, \tag{27}$$

where n=0 or 2, and integration is carried out over the first Brillouin zone, whose volume is $V_{\rm B}$. By the use of this approximation F_σ and F_κ in equation (8) are expressed as

$$\begin{aligned}
F_\sigma &= I_\sigma G^{(0)}(\gamma), & (28)\\
F_\kappa &= I_\sigma G^{(0)}(\gamma) + I_\kappa^{(2)} G^{(2)}(\gamma), & (29)\\
I_\sigma &= \int_{-1}^{\mu_{\rm max}} d\mu \frac{e^{-2W(q)} \mid f(q) \mid^2}{\mid \varepsilon(q,0) \mid^2}(1-\beta^2 q^2), & (30)\\
I_\kappa^{(2)} &= \int_{-1}^{\mu_{\rm max}} d\mu \frac{e^{-2W(q)} \mid f(q) \mid^2}{q^2 \mid \varepsilon(q,0) \mid^2}(1-\beta^2 q^2)\left(-\frac{1}{2}q^2+\frac{3}{4}\right), & (31)\\
q &= \left(\frac{1-\mu}{2}\right)^{1/2}, & (32)\\
q_{\rm min} &= \left(\frac{1-\mu_{\rm max}}{2}\right)^{1/2}, & (33)\\
\mu_{\rm max} &= 1-0.3575Z^{-2/3}. & (34)
\end{aligned}$$

Here we have introduced a small momentum transfer cutoff $q_{\rm min}$ corresponding to the unavailability of *Umklapp* processes for $q < q_{\rm min}$. The contributions of the normal processes are very much smaller than those of the *Umklapp* processes. For the choise of $q_{\rm min}$ we have followed Raikh & Yakovlev (1982). Yakovlev & Urpin

(1981) derived the asymptotic expressions of $G^{(0)}(\gamma)$ and $G^{(2)}(\gamma)$ for $\gamma \ll 1$ and $\gamma \gg 1$, and proposed the following analytic formulae for arbitrary γ , which fit the main terms of the asymptotic expressions:

$$G^{(0)}(\gamma) = u_{-2}\left[1 + \left(\frac{3u_{-2}\gamma}{\pi^2 c_2}\right)^2\right]^{-1/2} \approx 13.00(1 + 0.0174\gamma^2)^{-1/2}, \tag{35}$$

$$G^{(2)}(\gamma) = \frac{\gamma^2}{\pi^2}\left[1 + \left(\frac{15}{4\pi^4 c_2}\right)^{2/3}\gamma^2\right]^{-3/2} = \frac{\gamma^2}{\pi^2}(1 + 0.0118\gamma^2)^{-3/2}, \tag{36}$$

where $u_{-2} \approx 13.00$ (Pollock & Hansen 1973) and $c_2 = 29.98$ (Coldwell-Horsfall & Maradudin 1960) are the numerical constants that are characteristic of the phonon spectrum of the bcc Coulomb lattice. Raikh & Yakovlev (1982) calculated $G^{(0)}(\gamma)$ and $G^{(2)}(\gamma)$ numerically with the exact spectrum of phonons for $\gamma < 100$. It has been confirmed that the fitting formulae (35) and (36) have an accuracy better than 10 % even at $\gamma \sim 1$.

We have carried out the numerical integrations of equations (30) and (31) for ^{4}He, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S, ^{40}Ca, and ^{56}Fe. Some of the results are presented in Figures 1–6. For comparison we have also included the case where we have neglected the effects of the Debye - Waller factor and set $e^{-2W} = 1$. We also show the results of Raikh & Yakovlev (1982), which are

$$[I_\sigma]_{\rm RY} = 2 - \beta^2, \tag{37}$$

$$[I_\kappa^{(2)}]_{\rm RY} = \ln Z - \beta^2 + 1.583. \tag{38}$$

It is readily seen that the Debye - Waller factor reduces the resistivities (enhances the conductivities) by a factor of 2 – 4 near the melting temperature.

In Figures 7 – 9 we show the thermal conductivities at the crystallization point $\Gamma = 180$ as functions of densities. Thermal conductivities in the crystalline lattice phase are the present results. Thermal conductivities in the liquid metal phase have been calculated by using the method of Itoh et al. (1983). It is found that the thermal conductivity in the crystalline lattice phase is generally higher than that in the liquid metal phase by a factor 2 – 4 at the crystallization point $\Gamma = 180$.

In Figures 10 – 13 we show the thermal conductivity of the "pure" ^{12}C matter as a function of the parameter Γ for the densities 10^4 g cm^{-3}, 10^6 g cm^{-3} , 10^8 g cm^{-3}, and 10^{10} g cm^{-3}. The thermal conductivity in the crystalline lattice phase is the present result, and the thermal conductivity in the liquid metal phase has been calculated by using the method of Itoh et al. (1983). The end point of the curve in the crystalline phase corresponds to the condition (40). We emphasize that in practice impurity scattering dominates over phonon scattering at low enough temperatures (Itoh & Kohyama 1993). Therefore, the thermal conductivity in the crystalline phase presented here is an "ideal" thermal conductivity.

For the convenience of applications we have fitted the numerical results of the calculation by analytic formulae. We introduce the following variable:

$$u = 2\pi(\log_{10}\rho)/25.6\ . \tag{39}$$

The fitting has been carried out for the ranges $10^{0.0} \le \rho \le 10^{12.7}$ g cm^{-3}, $180 \le \Gamma \le 5000$. For the parameters $\Gamma > 5000$ the numerical result remains practically the same as in the case $\Gamma = 5000$. However, we note the condition

$$\gamma^{-1} \ge Z^{1/3}\frac{e^2}{\hbar v_{\rm F}} = \frac{Z^{1/3}}{137.0}\left\{1 + \frac{1}{1.018[(Z/A)\rho_6]^{2/3}}\right\}^{1/2} \tag{40}$$

must hold in order that the *Umklapp* processes remain effective (Yakovlev & Urpin 1981; Raikh & Yakovlev 1982).

The fitting formulae are taken as follows:

$$I_\sigma(u,\Gamma) = (1-v)I_\sigma(u,180) + vI_\sigma(u,5000), \tag{41}$$

$$I_\kappa^{(2)}(u,\Gamma) = (1-w)I_\kappa^{(2)}(u,180) + wI_\kappa^{(2)}(u,5000), \tag{42}$$

$$I_\sigma(u,180) = \sum_{m=1}^{10} a_m \sin mu + bu + c, \tag{43}$$

$$I_\sigma(u,5000) = \sum_{m=1}^{10} d_m \sin mu + eu + f, \tag{44}$$

$$I_\kappa^{(2)}(u,180) = \sum_{m=1}^{10} g_m \sin\, mu + hu + i, \tag{45}$$

$$I_\kappa^{(2)}(u,5000) = \sum_{m=1}^{10} j_m \sin\, mu + ku + l, \tag{46}$$

$$v = \sum_{m=0}^{3} \alpha_m \Gamma^{-m/3}, \tag{47}$$

$$w = \sum_{m=0}^{3} \beta_m \Gamma^{-m/3}. \tag{48}$$

The coefficients are given in Tables 1 – 3. The overall accuracy of the fitting is better than 6%.

4. CONCLUDING REMARKS

We have extended the calculation of Itoh et al. (1984a) to lower densities, and we have obtained a complete set of results of the thermal conductity in the crystalline lattice phase due to phonon scattering for the density range $10^{0.0} - 10^{12.7}$ g cm^{-3}. Comparing with the results of the calculation of the thermal conductity in the liquid metal phase reported in the paper of Itoh et al. (1983), we have shown that the thermal conductivity in the crystalline lattice phase is generally higher than that in the liquid metal phase by a factor 2 - 4 at the crystallization point $\Gamma = 180$.

For the sake of applications we have given the results in the form of analytic fitting formulae. The thermal conductivity, for instance, is given by equations (6) and (7), where the terms in formula (29) for F_κ are to be evaluated by using the fitting formulae (41) - (48) with the coefficients, for the appropriate composition, taken from Tables 1 - 3. The fitting formulae (35) and (36) are also to be used. This form for the conductivity is valid for $10^{0.0} \leq \rho \leq 10^{12.7}$ g cm^{-3} and $\Gamma \geq 180$. Condition (40) together with the conditions (2) and (3) must hold. The case of the neutron star matter has been already calculated in the paper of Itoh et al. (1984a) with an erratum. Impurity scattering contributions to the electrical and thermal conductivities of dense matter in the crystalline lattice phase have been calculated by Itoh & Kohyama (1993).

Our calculation covers almost all the elements of astrophysical importance. The present result will complement the published data of the electrical and thermal conductivities of dense matter and will contribute to the studies of stellar evolution and asteroseismology.

TABLE 1

COEFFICIENTS IN THE FITTING FORMULAE FOR $I_\sigma(u,180)$ AND $I_\sigma(u,5000)$

Coefficient	^{4}He	^{12}C	^{16}O	^{20}Ne	^{24}Mg	^{28}Si	^{32}S	^{40}Ca	^{56}F
a_1	0.50472	0.50998	0.45937	0.41952	0.38958	0.36656	0.34807	0.31920	0.288
a_2	0.31932	0.16533	0.13914	0.12145	0.10717	0.09486	0.08411	0.06649	0.046
a_3	0.04577	0.04904	0.04859	0.04525	0.04160	0.03838	0.03562	0.03119	0.025
a_4	-0.03333	-0.04479	-0.04650	-0.04549	-0.04362	-0.04164	-0.03980	-0.03659	-0.033
a_5	-0.01113	-0.00038	0.00330	0.00500	0.00568	0.00593	0.00600	0.00593	0.005
a_6	0.01983	0.01222	0.00760	0.00455	0.00259	0.00124	0.00025	-0.00109	-0.002
a_7	0.00503	0.00235	0.00348	0.00421	0.00451	0.00458	0.00455	0.00435	0.003
a_8	-0.00578	-0.00646	-0.00683	-0.00703	-0.00698	-0.00680	-0.00657	-0.00607	-0.005
a_9	-0.00247	-0.00054	0.00025	0.00084	0.00119	0.00137	0.00147	0.00152	0.001
a_{10}	0.00241	0.00134	0.00057	-0.00003	-0.00043	-0.00068	-0.00084	-0.00101	-0.001
b	-0.04207	-0.01214	-0.00445	0.00028	0.00315	0.00492	0.00604	0.00727	0.008
c	0.58334	0.40868	0.35433	0.31167	0.27718	0.24868	0.22473	0.18681	0.143
d_1	0.49119	0.71734	0.74303	0.75895	0.77063	0.78011	0.78826	0.80205	0.827
d_2	0.38841	0.36575	0.35539	0.34558	0.33617	0.32720	0.31874	0.30345	0.283
d_3	0.08448	0.08528	0.08781	0.09004	0.09229	0.09466	0.09712	0.10213	0.106
d_4	-0.02659	-0.06504	-0.07463	-0.08157	-0.08675	-0.09073	-0.09390	-0.09877	-0.105
d_5	-0.01187	-0.00514	-0.00156	0.00169	0.00462	0.00720	0.00943	0.01298	0.018
d_6	0.02200	0.02633	0.02541	0.02439	0.02335	0.02229	0.02126	0.01937	0.017
d_7	0.00745	0.00659	0.00737	0.00823	0.00898	0.00963	0.01020	0.01113	0.011
d_8	-0.00521	-0.00986	-0.01112	-0.01214	-0.01298	-0.01366	-0.01420	-0.01498	-0.015
d_9	-0.00252	-0.00156	-0.00089	-0.00030	0.00022	0.00067	0.00105	0.00162	0.002
d_{10}	0.00275	0.00319	0.00292	0.00263	0.00237	0.00213	0.00193	0.00162	0.001
e	-0.05892	-0.04356	-0.03889	-0.03583	-0.03382	-0.03251	-0.03167	-0.03088	-0.029
f	0.79919	0.82040	0.81777	0.81431	0.81071	0.80715	0.80369	0.79710	0.777

TABLE 2

COEFFICIENTS IN THE FITTING FORMULAE FOR $I_\kappa^{(2)}(u, 180)$ AND $I_\kappa^{(2)}(u, 5000)$

Coefficient	^{4}He	^{12}C	^{16}O	^{20}Ne	^{24}Mg	^{28}Si	^{32}S	^{40}Ca	^{56}F
g_1	1.07772	1.38810	1.37409	1.36240	1.35811	1.35886	1.36196	1.36931	1.373
g_2	0.36016	0.20019	0.17862	0.16060	0.14141	0.12112	0.10065	0.06149	0.003
g_3	0.03619	0.05281	0.03992	0.02357	0.00858	-0.00414	-0.01486	-0.03197	-0.055
g_4	-0.04081	-0.11228	-0.12966	-0.13902	-0.14482	-0.14899	-0.15222	-0.15686	-0.160
g_5	-0.02063	-0.01393	-0.00947	-0.00717	-0.00570	-0.00433	-0.00287	0.00029	0.006
g_6	0.01423	-0.00360	-0.00999	-0.01356	-0.01545	-0.01653	-0.01719	-0.01790	-0.017
g_7	-0.00137	0.00065	0.00405	0.00654	0.00822	0.00945	0.01044	0.01199	0.013
g_8	-0.00820	-0.01419	-0.01599	-0.01710	-0.01765	-0.01791	-0.01806	-0.01822	-0.018
g_9	-0.00422	-0.00047	0.00139	0.00280	0.00373	0.00437	0.00484	0.00551	0.006
g_{10}	0.00148	-0.00140	-0.00277	-0.00382	-0.00454	-0.00505	-0.00542	-0.00596	-0.006
h	-0.03114	0.05435	0.07973	0.09677	0.10835	0.11647	0.12239	0.13037	0.138
i	0.57026	0.48610	0.44848	0.41645	0.38895	0.36506	0.34410	0.30897	0.261
j_1	1.10089	1.72569	1.83163	1.91047	1.97509	2.03074	2.08002	2.16498	2.269
j_2	0.42911	0.37806	0.35808	0.33682	0.31486	0.29284	0.27119	0.22991	0.166
j_3	0.06803	0.06998	0.06153	0.05360	0.04700	0.04177	0.03774	0.03239	0.021
j_4	-0.03627	-0.12770	-0.15298	-0.17229	-0.18767	-0.20033	-0.21106	-0.22858	-0.250
j_5	-0.02189	-0.02128	-0.01820	-0.01446	-0.01037	-0.00622	-0.00215	0.00554	0.018
j_6	0.01521	0.00428	0.00045	-0.00243	-0.00477	-0.00681	-0.00862	-0.01173	-0.015
j_7	-0.00017	0.00197	0.00475	0.00747	0.00995	0.01220	0.01424	0.01782	0.021
j_8	-0.00815	-0.01730	-0.01978	-0.02179	-0.02352	-0.02504	-0.02638	-0.02869	-0.031
j_9	-0.00445	-0.00173	-0.00011	0.00131	0.00256	0.00366	0.00463	0.00626	0.008
j_{10}	0.00158	-0.00043	-0.00148	-0.00243	-0.00328	-0.00405	-0.00474	-0.00593	-0.007
k	-0.04237	0.03393	0.05890	0.07726	0.09121	0.10212	0.11085	0.12387	0.138
l	0.71437	0.77500	0.77906	0.78017	0.77999	0.77918	0.77802	0.77519	0.754

TABLE 3

COEFFICIENTS IN THE FITTING FORMULAE FOR v AND w

Coefficient	^{4}He	^{12}C	^{16}O	^{20}Ne	^{24}Mg
α_0	1.6217	0.4481	0.4379	0.4990	0.5601
α_1	-22.7740	17.3468	19.0030	18.0483	16.8431
α_2	272.2686	-146.8104	-180.0697	-184.0299	-182.3298
α_3	-1103.0256	195.1610	332.0491	373.8639	391.6884
β_0	1.6467	0.5893	0.5815	0.6348	0.6860
β_1	-23.5974	12.5030	13.8645	12.9559	11.8861
β_2	280.4861	-95.9862	-123.5989	-125.3504	-122.5496
β_3	-1127.6517	37.1996	151.1576	180.4306	189.5117

Coefficient	^{28}Si	^{32}S	^{40}Ca	^{56}Fe
α_0	0.6086	0.6453	0.6964	0.7528
α_1	15.8721	15.1613	14.2469	13.2729
α_2	-180.6363	-179.9249	-180.5964	-182.1237
α_3	404.3601	416.3911	440.1475	469.6787
β_0	0.7244	0.7512	0.7824	0.8055
β_1	11.0727	10.5274	9.9749	9.7041
β_2	-120.1939	-119.0817	-119.6607	-123.2175
β_3	195.2377	201.5125	216.7741	241.3355

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FIGURE LEGENDS

FIG.1. I_σ for the ^{4}He matter. RY stands for the results of Raikh & Yakovlev (1982).

FIG.2. $I_\kappa^{(2)}$ for the ^{4}He matter.

FIG.3. I_σ for the ^{12}C matter.

FIG.4. $I_\kappa^{(2)}$ for the ^{12}C matter.

FIG.5. I_σ for the ^{56}Fe matter.

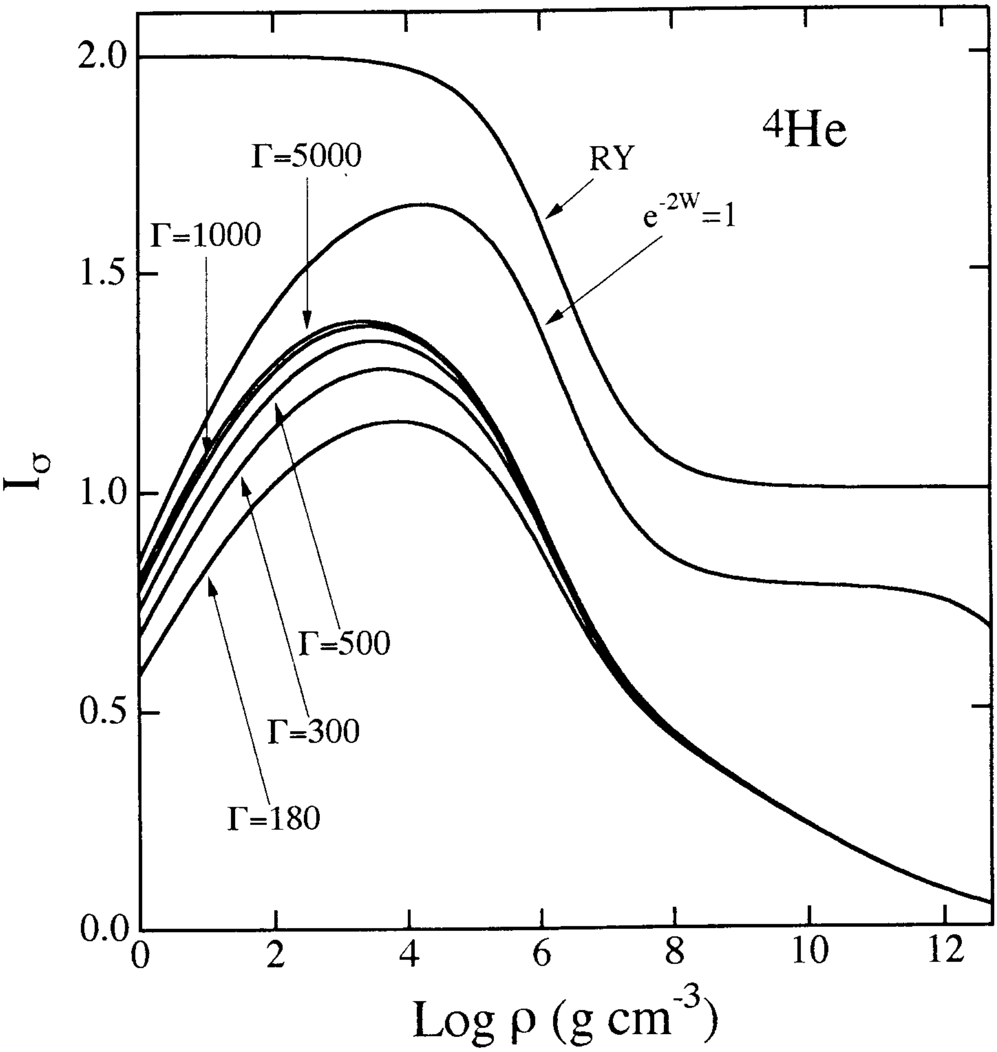
FIG.6. $I_\kappa^{(2)}$ for the ^{56}Fe matter.

FIG.7. Thermal conductivities of the ^{4}He matter at the crystallization point $\Gamma = 180$.

FIG.8. Thermal conductivities of the ^{12}C matter at the crystallization point $\Gamma = 180$.

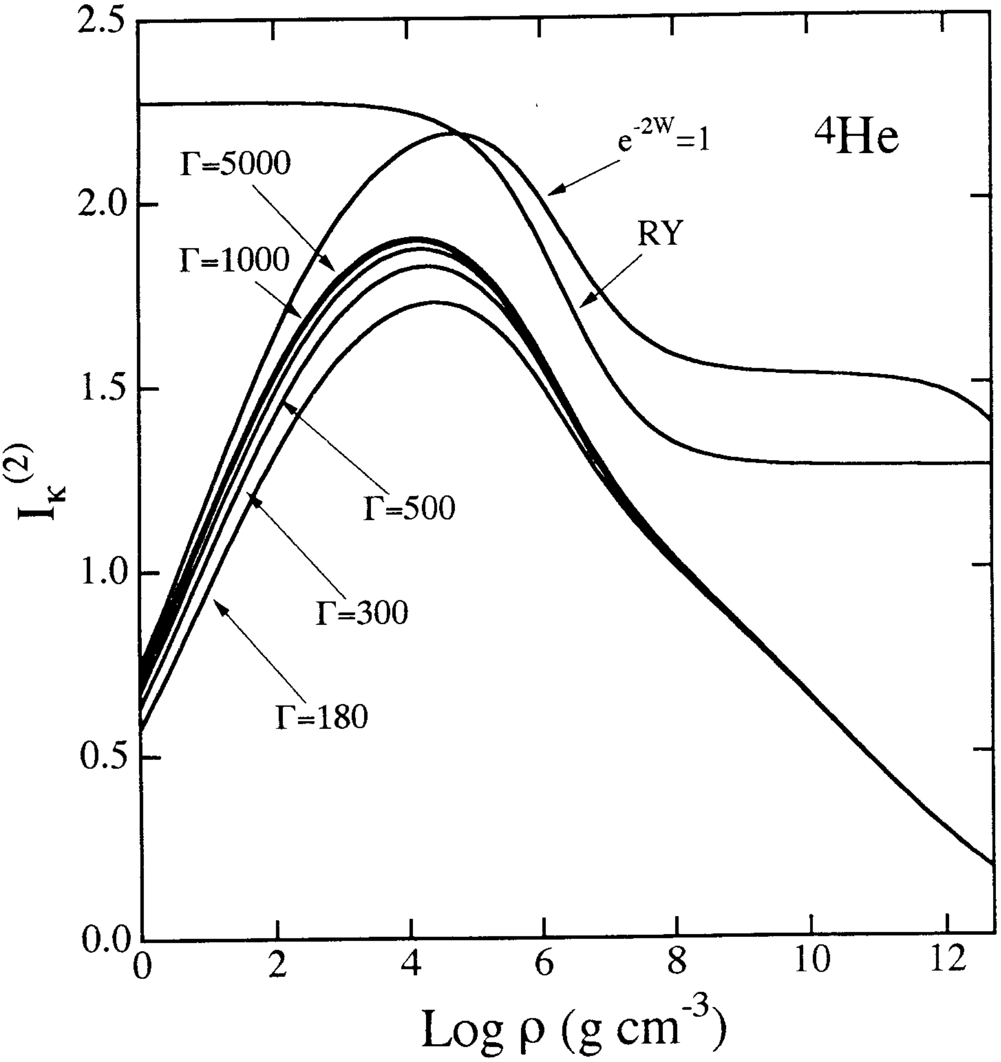
FIG.9. Thermal conductivities of the ^{56}Fe matter at the crystallization point $\Gamma = 180$.

FIG.10. Thermal conductivity of the "pure" ^{12}C matter as a function of the parameter Γ for the densities 10^4 g cm^{-3}, 10^6 g cm^{-3}, 10^8 g cm^{-3}, 10^{10} g cm^{-3}.



^{4}He
RY
$e^{-2W}=1$
$\Gamma=5000$
$\Gamma=1000$
$\Gamma=500$
$\Gamma=300$
$\Gamma=180$
I_σ
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
12
Log ρ (g cm^{-3})

FIG.1



^{4}He
$e^{-2W}=1$
RY
Γ=5000
Γ=1000
Γ=500
Γ=300
Γ=180
$I_\kappa^{(2)}$
Log ρ (g cm^{-3})
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
12

FIG.2

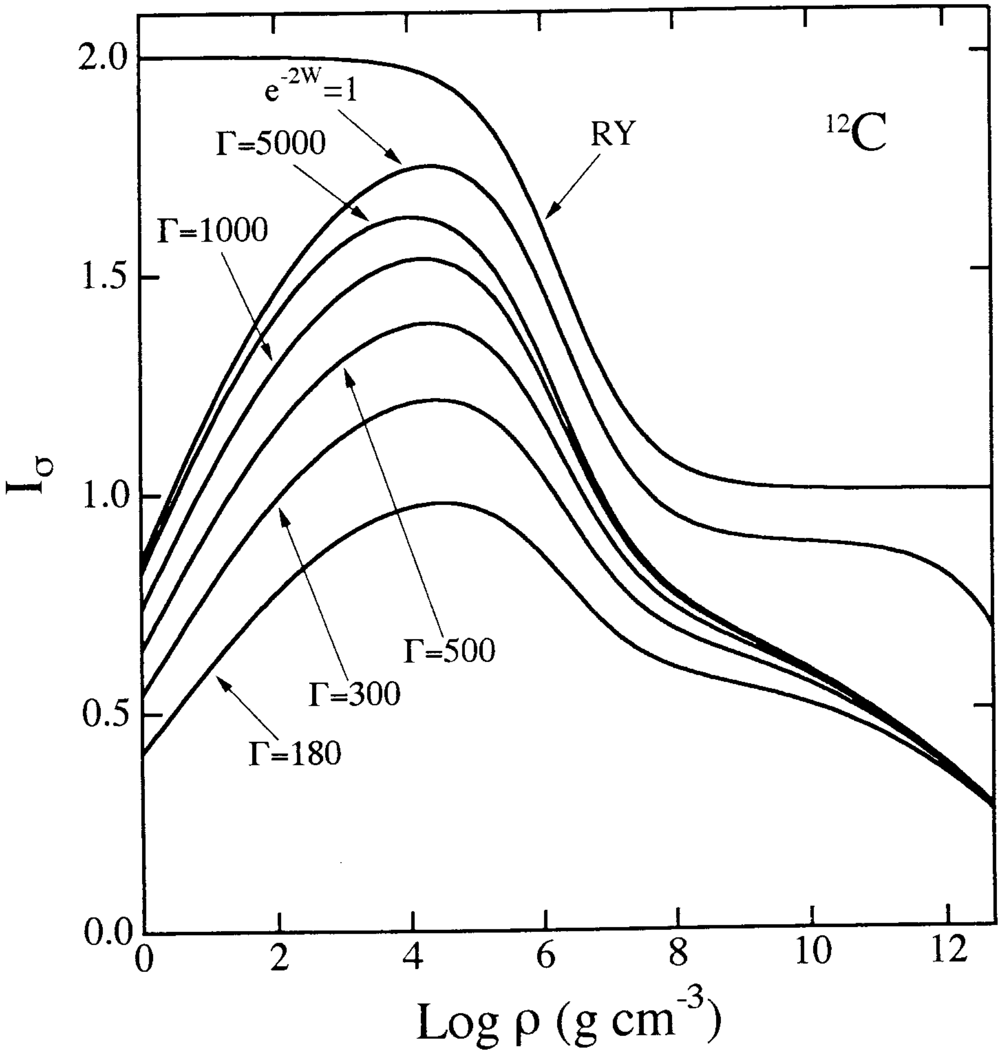

I_σ
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
12
Log ρ (g cm^{-3})
$e^{-2W}=1$
Γ=5000
Γ=1000
Γ=500
Γ=300
Γ=180
RY
^{12}C

FIG.3

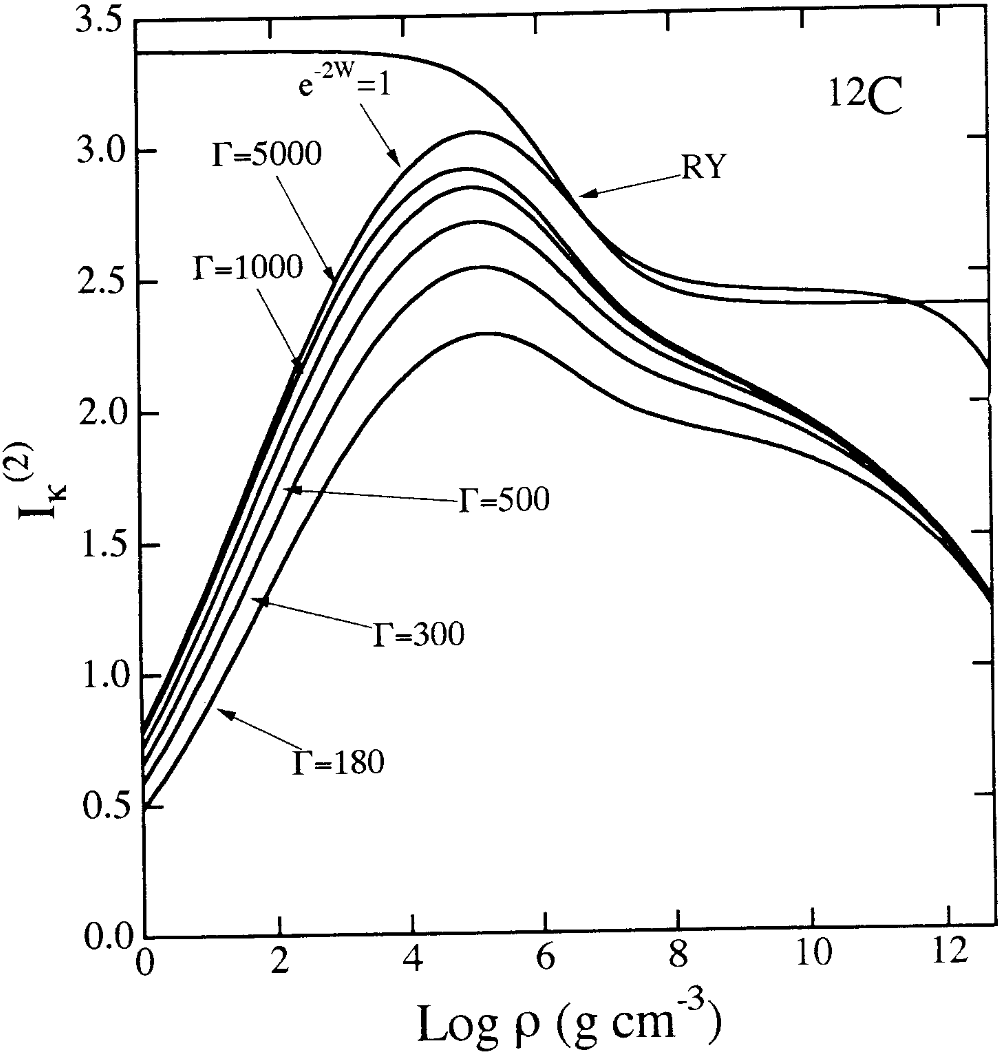

3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
12
$I_\kappa^{(2)}$
Log ρ (g cm^{-3})
12C
$e^{-2W}=1$
Γ=5000
Γ=1000
Γ=500
Γ=300
Γ=180
RY

FIG.4

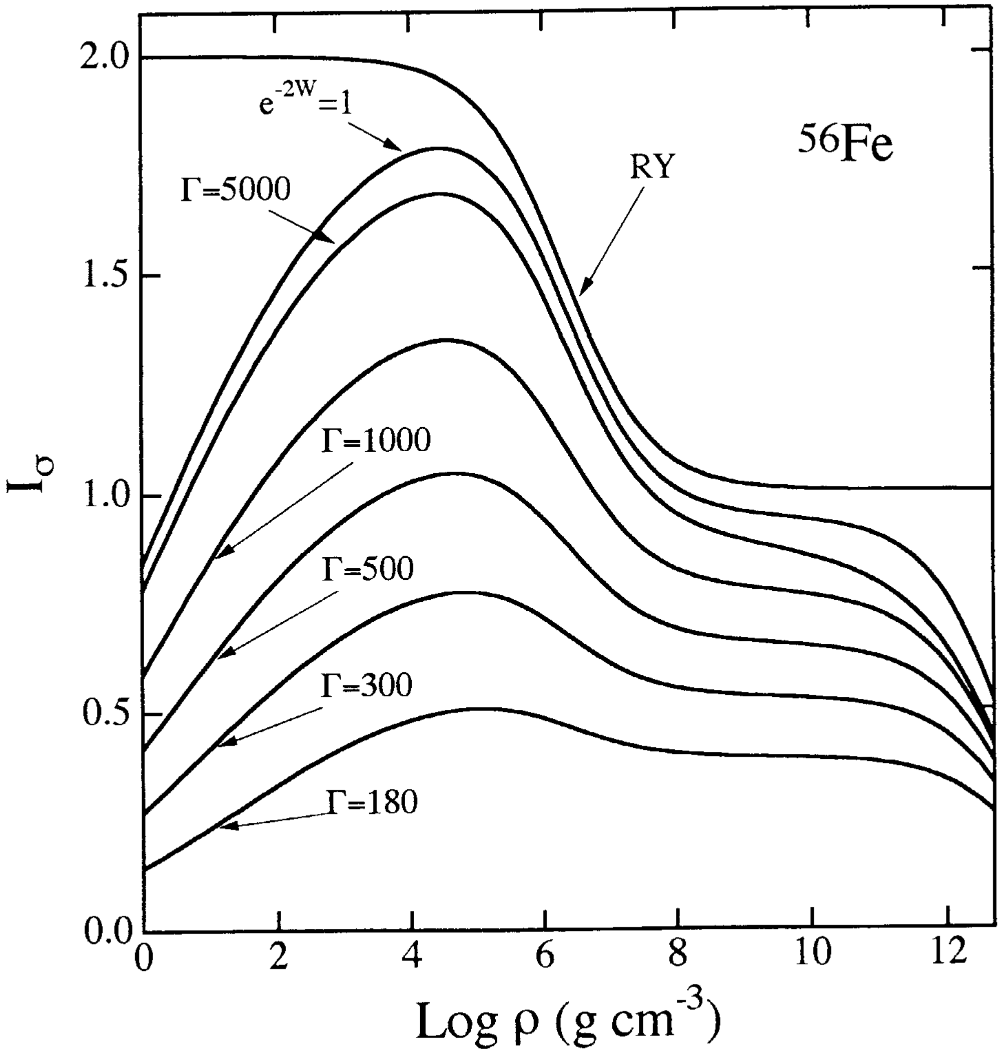
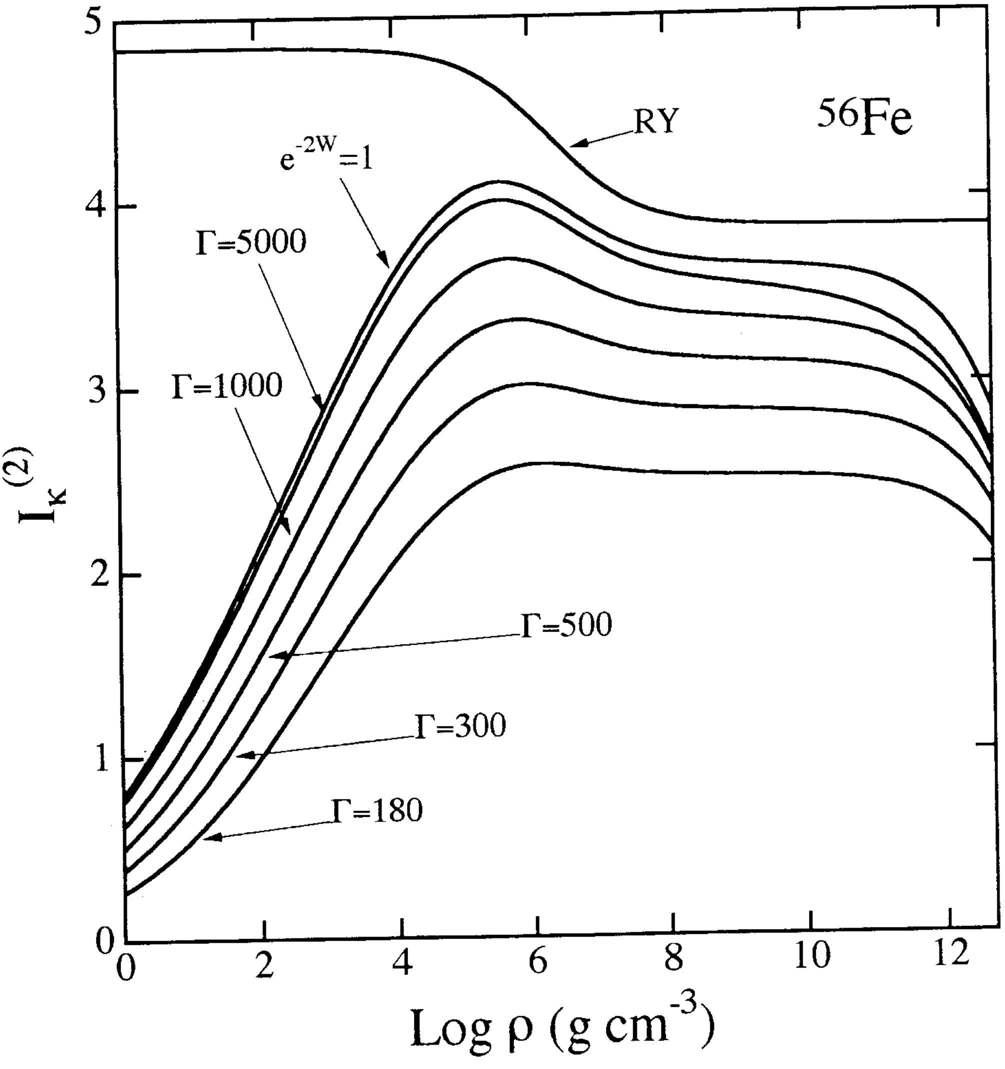

2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
12
I_σ
Log ρ (g cm^{-3})
$e^{-2W}=1$
Γ=5000
Γ=1000
Γ=500
Γ=300
Γ=180
RY
56Fe

FIG.5



5
4
3
2
1
0
0
2
4
6
8
10
12
$I_\kappa^{(2)}$
Log ρ (g cm^{-3})
^{56}Fe
RY
$e^{-2W}=1$
Γ=5000
Γ=1000
Γ=500
Γ=300
Γ=180

FIG.6

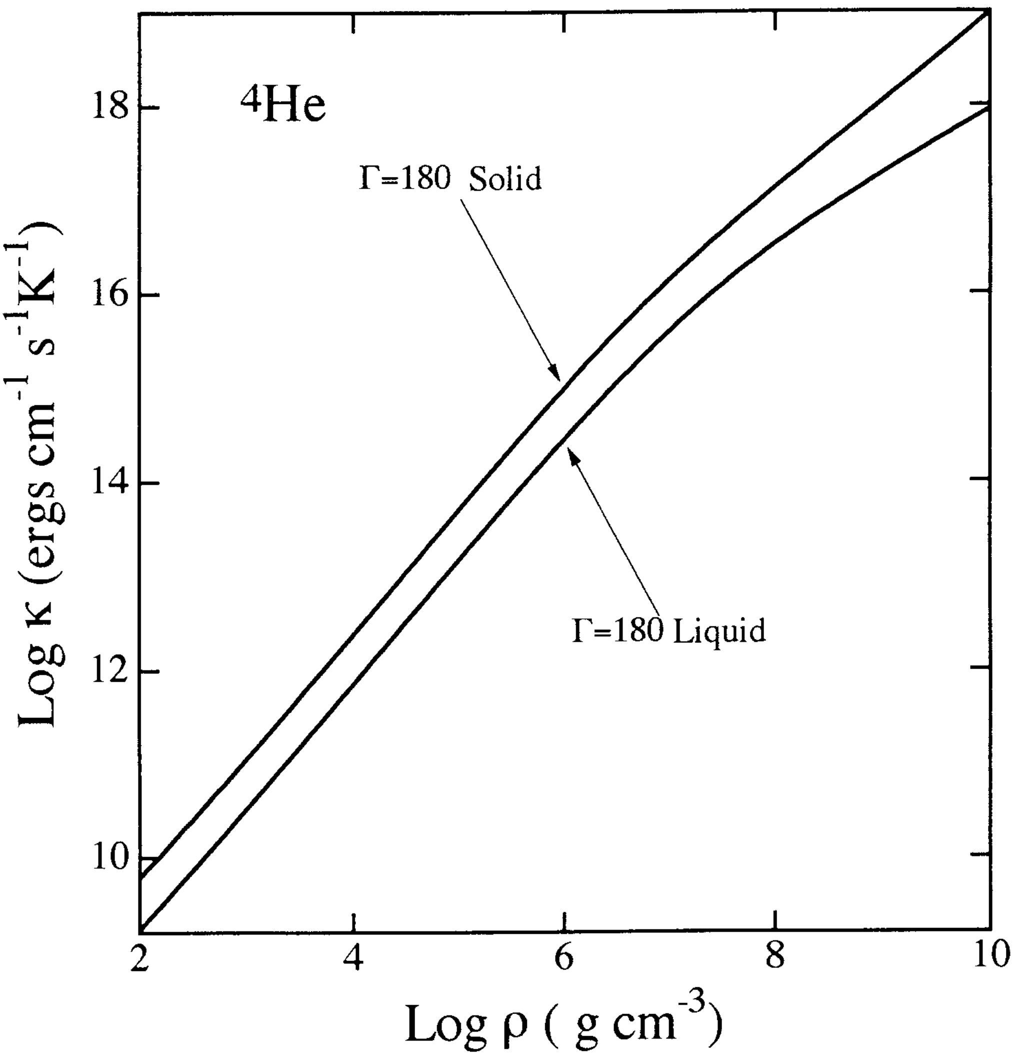
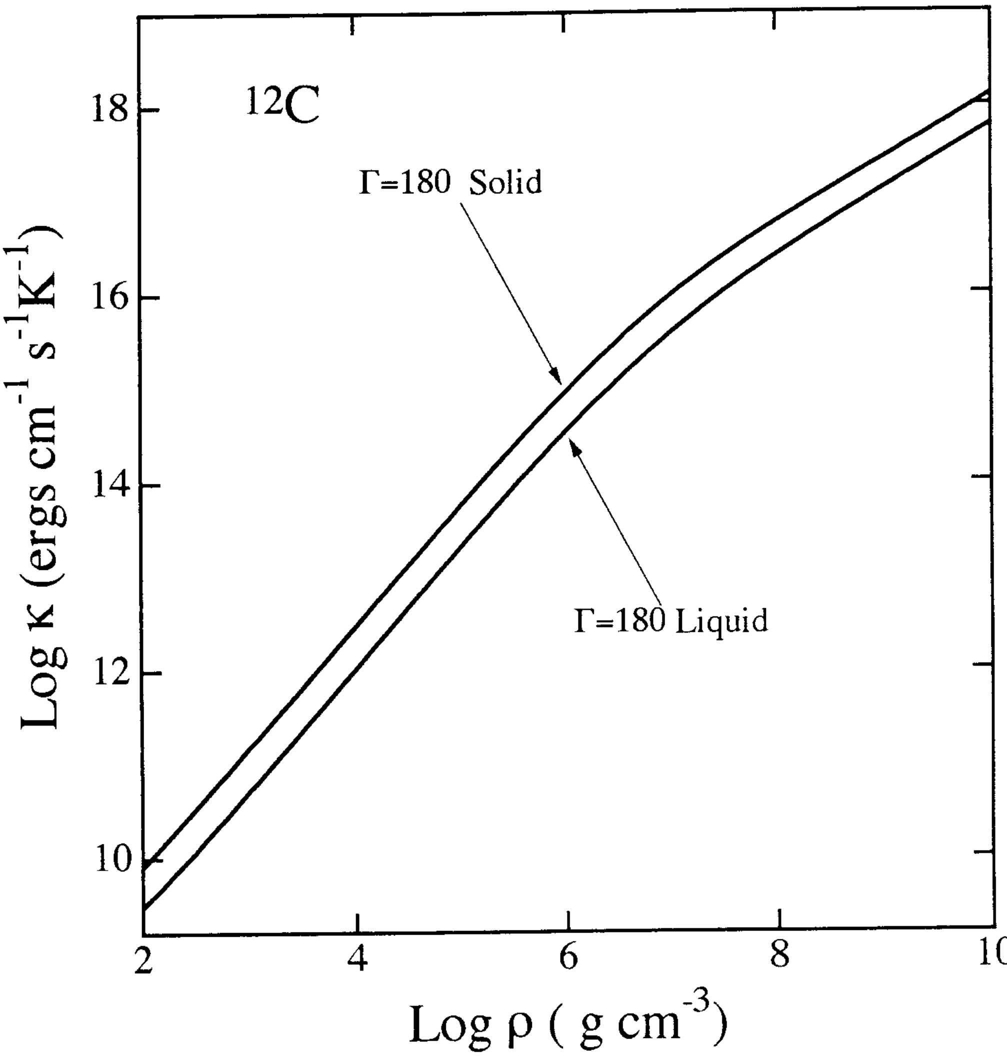

^{4}He
Γ=180 Solid
Γ=180 Liquid
Log κ (ergs cm^{-1} s^{-1}K^{-1})
10
12
14
16
18
2
4
6
8
10
Log ρ (g cm^{-3})

FIG.7



^{12}C
Γ=180 Solid
Γ=180 Liquid
18
16
14
12
10
Log κ (ergs cm^{-1} s^{-1}K^{-1})
2
4
6
8
10
Log ρ (g cm^{-3})

FIG.8

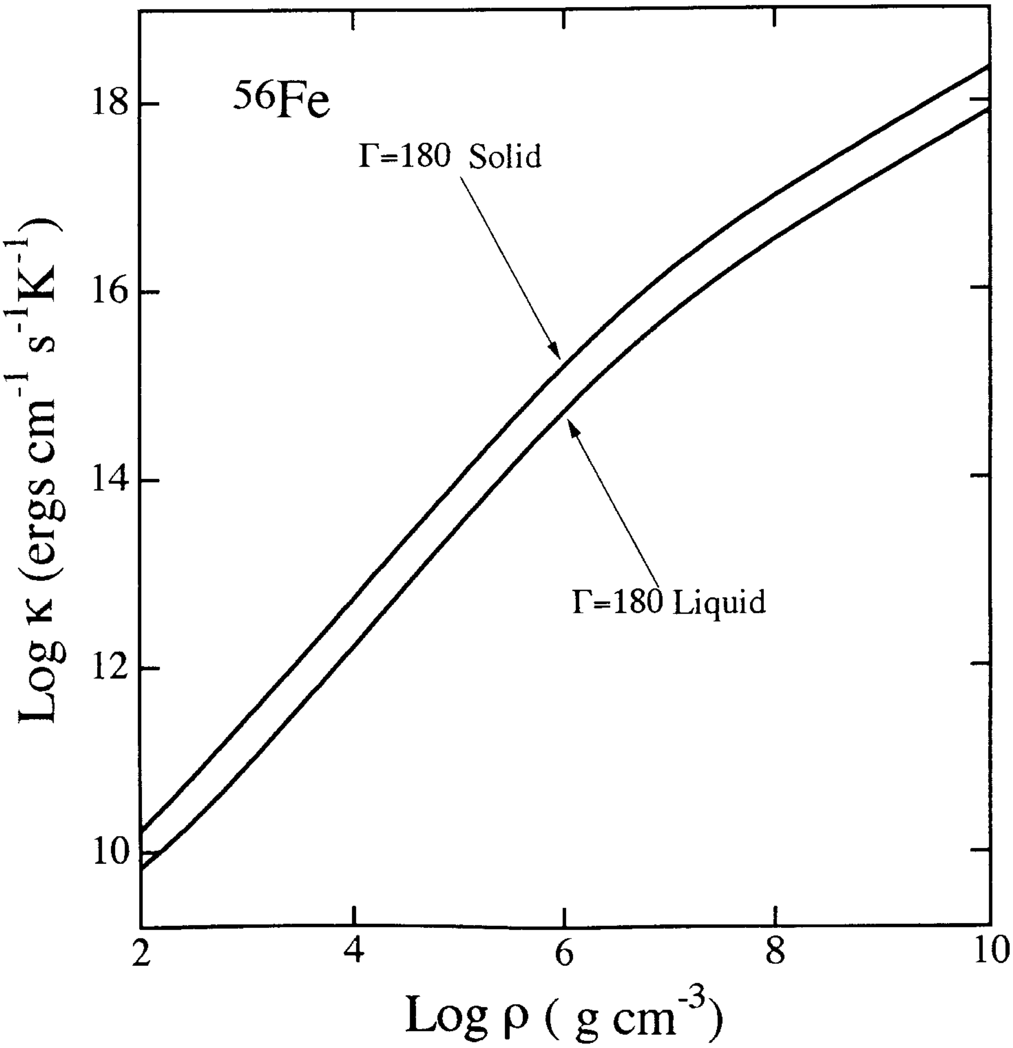

56Fe
Γ=180 Solid
Γ=180 Liquid
Log κ (ergs cm-1 s-1 K-1)
Log ρ (g cm-3)
10
12
14
16
18
2
4
6
8
10

FIG.9

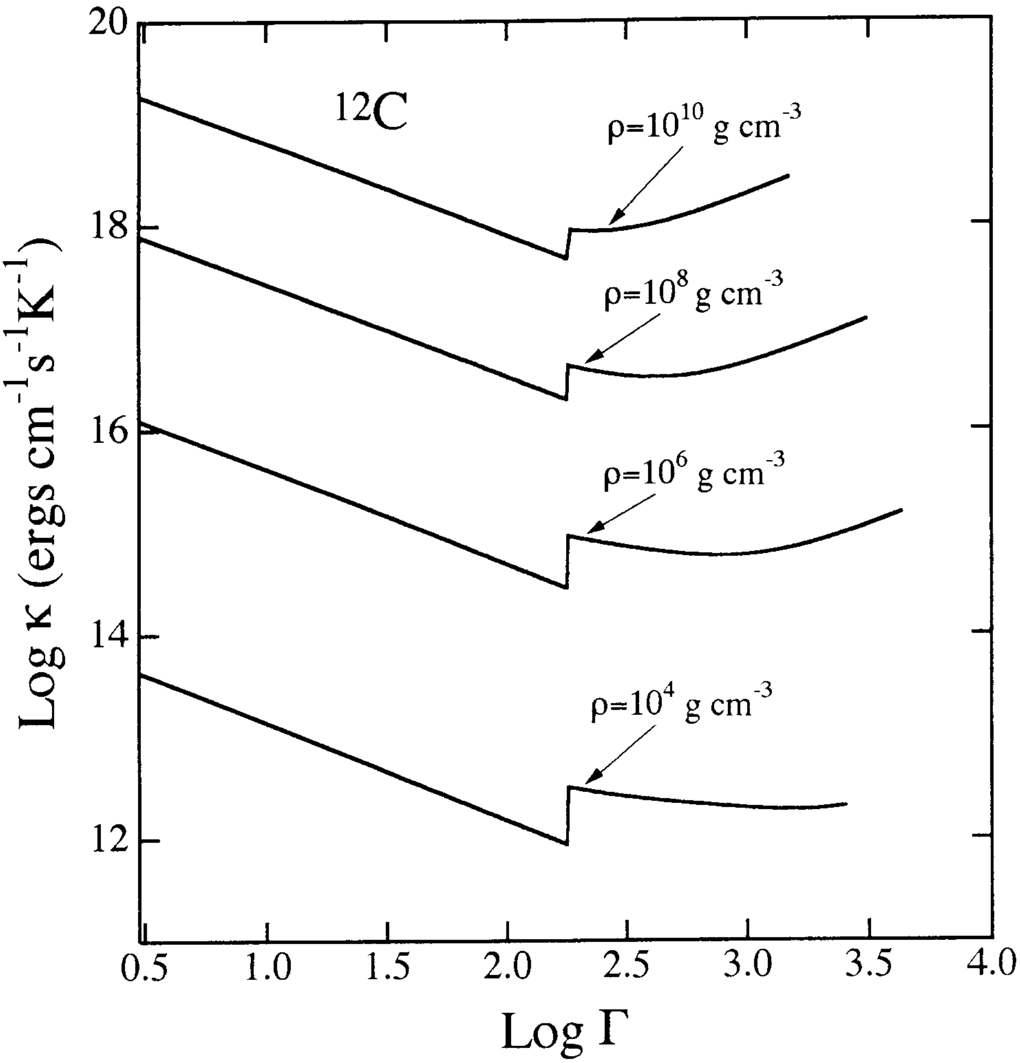

12C
ρ=10^10 g cm^-3
ρ=10^8 g cm^-3
ρ=10^6 g cm^-3
ρ=10^4 g cm^-3
Log κ (ergs cm^-1 s^-1 K^-1)
20
18
16
14
12
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Log Γ

FIG.10