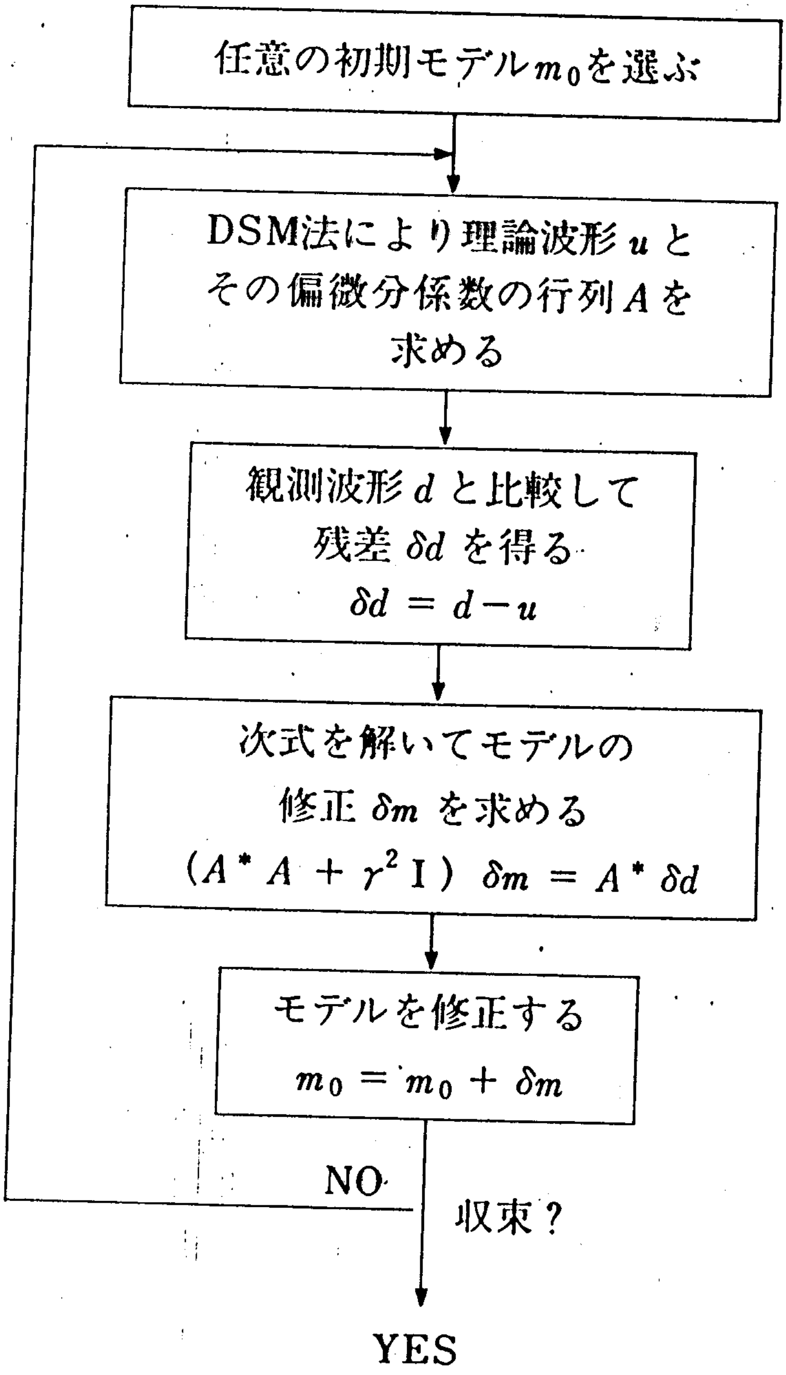
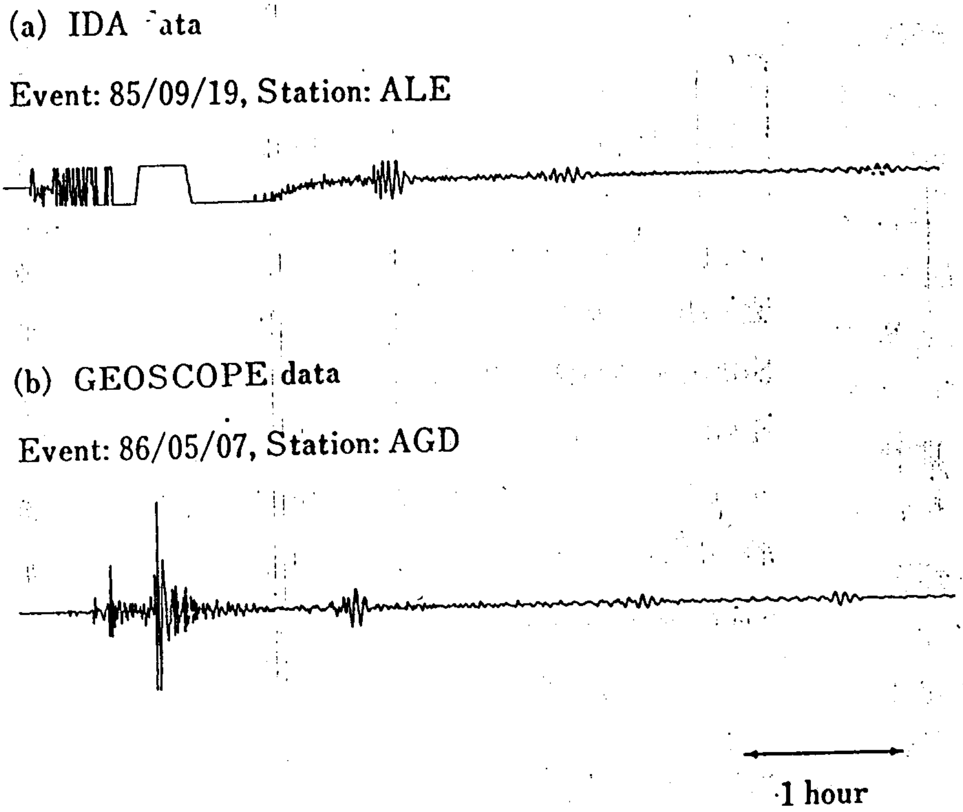

任意の初期モデル$m_0$を選ぶ
DSM法により理論波形$u$とその偏微分係数の行列$A$を求める
観測波形$d$と比較して残差$\delta d$を得る
$\delta d = d - u$
次式を解いてモデルの修正$\delta m$を求める
$(A^* A + \gamma^2 \mathrm{I})\ \delta m = A^* \delta d$
モデルを修正する
$m_0 = m_0 + \delta m$
NO
収束？
YES

図2　逐次線形波形インバージョンのフローチャート．

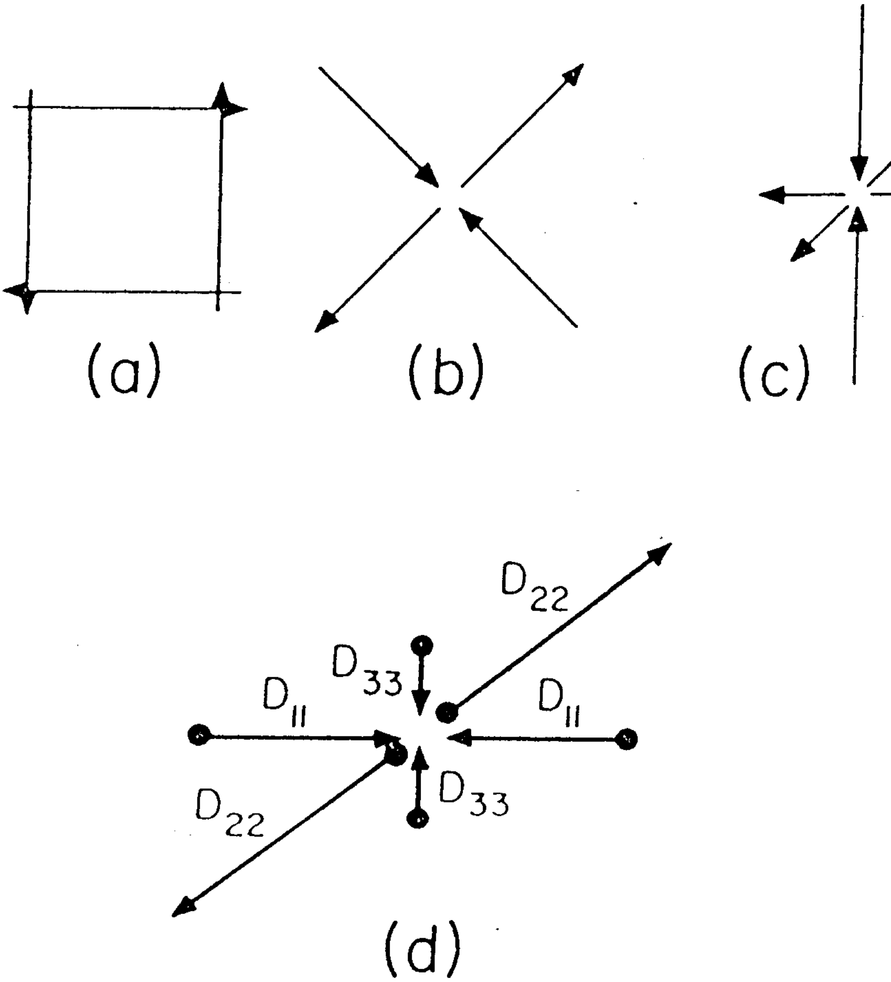

(a) IDA data

Event: 85/09/19, Station: ALE

(b) GEOSCOPE data

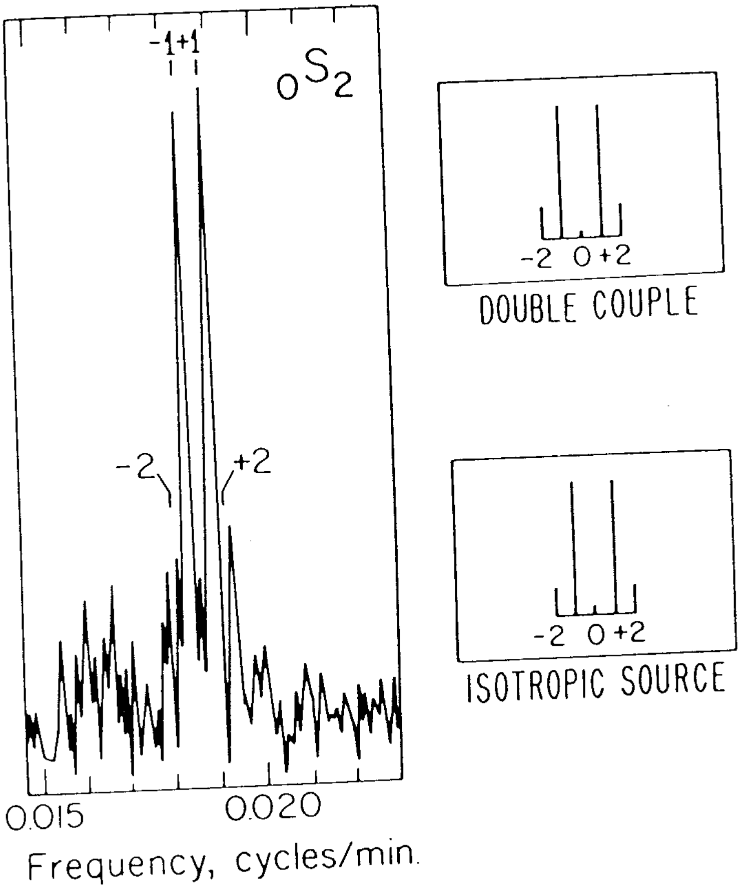
Event: 86/05/07, Station: AGD

図1　観測地震波形記録の例(6時間分)．(a)はIDA観測網の記録，(b)はGEOSCOPE観測網の記録．(a)では大振幅の波が到着した，記録の最初の部分が飽和していて使えないが，(b)では最初の部分もきれいに記録されている．地震の大きさは，(a)，(b)でほぼ同じである．

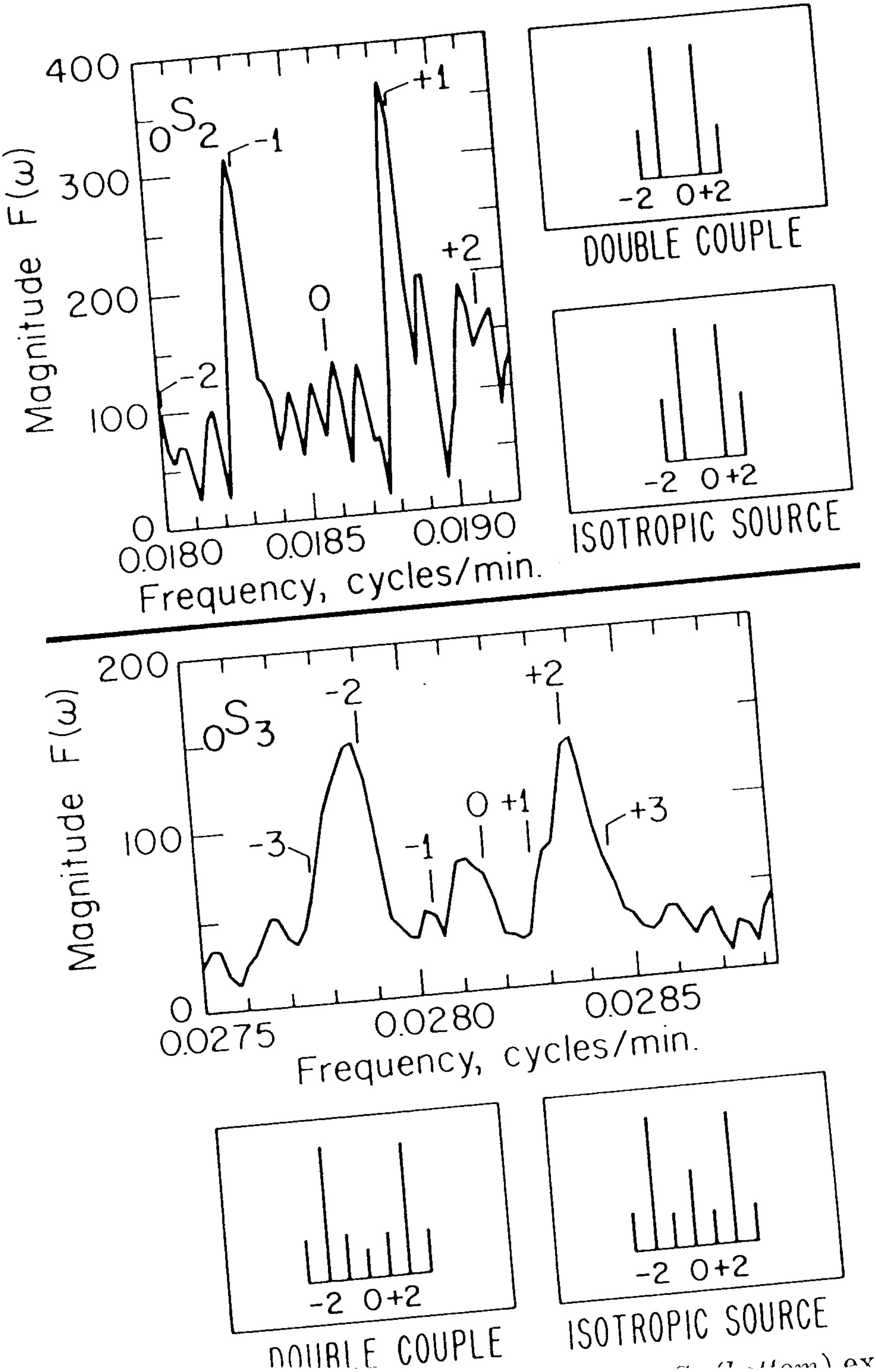

(a)
(b)
(c)
D22
D33
D11
D11
D33
D22
(d)

. Double couple, CLVD and triple doublet force systems: (a) and (b) The
nted two different ways. (c) The compensated linear vector dipole (CL
t," consisting of three mutually perpendicular force doublets with $D_{11} + D$

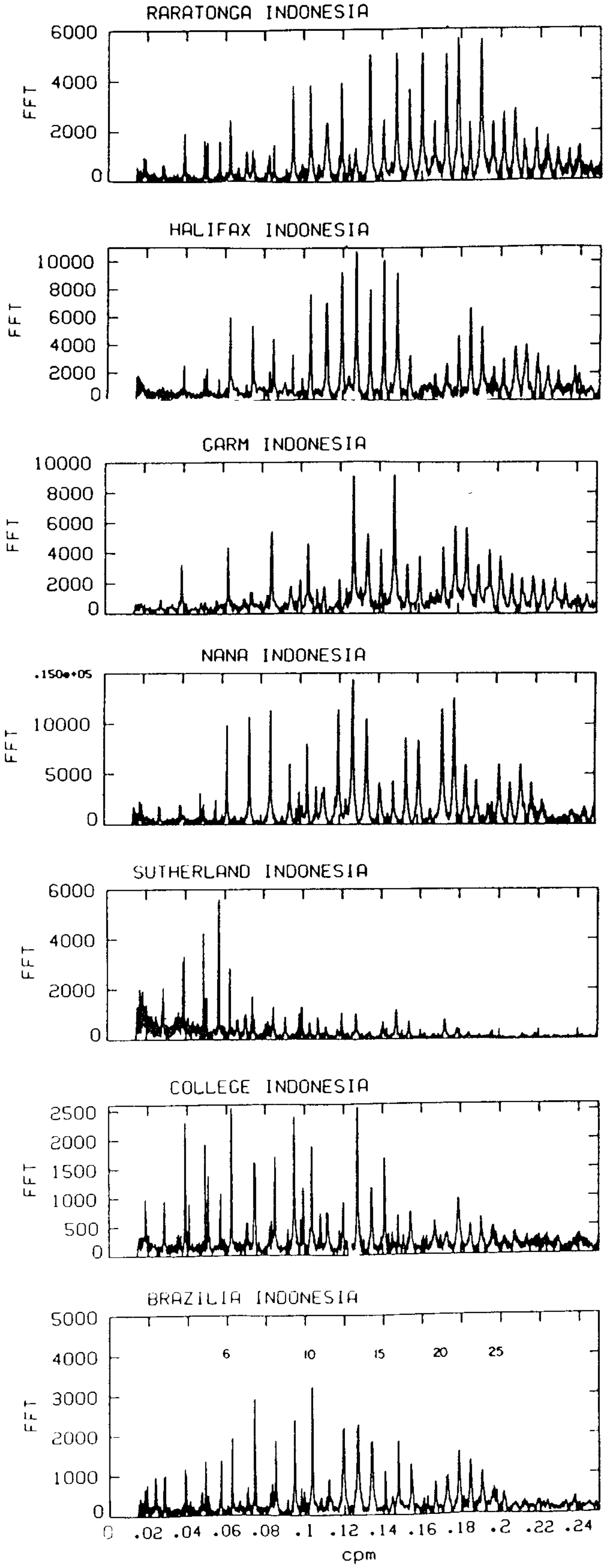
initeness has only a negligible effect on the relative spect
ows the spectrum from a gravity meter record (Slichter, 19
hquake, after processing and filtering by Wiggins and Mill



lit spheroidal mode spectra for $_0S_2$ excited by the Alaskan earthquak
meter at Los Angeles, California by Slichter (1967) after processing b
. Details are as in Figure 3. The finite fault geometry of Kanamori
le-couple source.


400
300
200
100
0
Magnitude F(ω)
$_0S_2$
-1
+1
+2
0
-2
0.0180
0.0185
0.0190
Frequency, cycles/min.
-2 0+2
DOUBLE COUPLE
-2 0+2
ISOTROPIC SOURCE
200
100
0
Magnitude F(ω)
$_0S_3$
-2
+2
-3
-1
0 +1
+3
0.0275
0.0280
0.0285
Frequency, cycles/min.
-2 0+2
DOUBLE COUPLE
-2 0+2
ISOTROPIC SOURCE

(bottom) exc


RARATONGA INDONESIA
FFT
6000
4000
2000
0
HALIFAX INDONESIA
FFT
10000
8000
6000
4000
2000
0
GARM INDONESIA
FFT
10000
8000
6000
4000
2000
0
NANA INDONESIA
FFT
.150e+05
10000
5000
0
SUTHERLAND INDONESIA
FFT
6000
4000
2000
0
COLLEGE INDONESIA
FFT
2500
2000
1500
1000
500
0
BRAZILIA INDONESIA
FFT
5000
4000
3000
2000
1000
0
6
10
15
20
25
0
.02
.04
.06
.08
.1
.12
.14
.16
.18
.2
.22
.24
cpm

# DIRECT SOLUTION METHOD

## Notation

- All results in frequency domain
- Cartesian coordinate system used
- （和の規約）for physical ($x$, $y$, or $z$) coordinates
- （和の規約）not used for vector space of trial functions, etc.
- Rotation and self-gravitation not included in this talk
- Explicit results for fluid layers not shown here.
- $\mathbf{u}(\mathbf{x})$ is the displacement
- $u_i$ is the $i$-component
- $\mathbf{x}$ is the spatial coordinate
- $\{\phi_i^{(1)}(\mathbf{x}), \ldots, \phi_i^{(N)}(\mathbf{x})\}$, are linearly independent vector *trial functions*

## Trial Function Expansion

$$u_i(\mathbf{x},\omega) = \sum_{n=1}^{N} c_n(\omega)\phi_i^{(n)}(\mathbf{x}), \tag{1}$$

- We hereafter write $u_i$ and $c_n$ rather than $u_i(\mathbf{x},\omega)$ and $c_n(\omega)$.
- Expansion coefficients, $c_n$, are the generalized coordinates.
- Each trial function must be continuous

# Weak form

The weak form of the elastic equation of motion is:

$$\omega^2 \int_V y_i^{(m)} \rho\, u_i \, dV - \int_V y_{i,j}^{(m)} C_{ijkl}\, u_{k,l} \, dV = - \int_V y_i^{(m)} f_i \, dV . \qquad (2)$$

$$(m = 1, \ldots, N)$$

- $\rho$ is the density
- $C_{ijkl} = C_{jikl} = C_{klij}$ are the 21 independent elastic constants.
- $C_{ijkl}$ includes anelastic attenuation is therefore complex and frequency-dependent.
- The subscript $_{,j}$ denotes spatial differentiation with respect to the $j$-coordinate
- $f_i$ is the $i$-component of the applied force.
- $y_i^{(m)}$ is a vector weight function.
- The displacement $u_i$ must be continuous.
- Continuity of the traction $(C_{ijkl} u_{k,l}) n_j$ in the $n_j$ direction is a *natural continuity condition* which is *automatically* satisfied by the weak form solution.
- We use the same number of weight functions and trial functions, so that the system of linear equations, Eq. (2), will have a unique solution.

## Equivalence of strong and weak solutions

We integrate Eq. (2) by parts:

$$\int_V y_i^{(m)} \left\{\rho\omega^2 u_i + (C_{ijkl} u_{k,l})_{,j} + f_i\right\} dV - \int_S y_i^{(m)} \{(C_{ijkl} u_{k,l}) n_j\} \, dS = 0 \qquad (3)$$

$$(m = 1, \ldots, N),$$

where $n_j$ is the outward unit normal vector.

The coefficients of the weight functions, $y_i^{(m)}$, in the volume integral in Eq. (3) – the terms in the braces – must be zero everywhere:

$$\left\{\rho\omega^2 u_i + (C_{ijkl} u_{k,l})_{,j} + f_i\right\} = 0. \qquad (4)$$

Even when the integration by parts cannot be rigorously performed, we can nevertheless regard this result as the limit of the result obtained for a sequence of admissible functions.

Similarly, the coefficients of the weight functions in the surface integral in Eq. (3) must also be zero everywhere:

$$\{(C_{ijkl} u_{k,l}) n_j\}. \qquad (5)$$

We thus see that a free surface boundary condition is automatically satsified without its having to be imposed explicitly.

The same type of derivation can be used to demonstrate that the weak form solutions automatically satisfy continuity of traction, but we do not present explicit results here. Also, by adding surface integrals to Eq. (2) we can alter the natural boundary conditions, but we also omit these results here.

### Finite approximation

The above discussion assumed that the trial and weight functions were complete sets, but all calculations will be limited to finite bases, which, in general, will not be complete sets. However, in the limit as the basis becomes infinite, the system of linear equations (Eq. 2 converges to the strong form solution.

## Strong form vs. weak form

- Boundary Conditions
  - **Strong Form**: Must be explicitly specified.
  - **Weak Form**: Included in operator. Natural boundary conditions are automatically satisfied. To change b.c., must add (or change) surface integral.
- Continuity Conditions
  - **Strong Form**: Continuity of displacement and traction must be enforced.
  - **Weak Form**: Continuity of displacement must be enforced, but continuity of traction is automatically satisfied.

## DIRECT SOLUTION METHOD

We hereafter use the same functions as both weight functions and trial functions: $y_i^{(m)} = (\phi_i^{(m)})^*$ . We use these weight functions in Eq. (2) to obtain the *Galerkin weak form* of the equation of motion:

$$\left(\omega^2 \mathbf{T} - \mathbf{H}\right) \mathbf{c} = -\mathbf{g}, \tag{6}$$

- $\mathbf{T}$ is the mass matrix
- $\mathbf{H}$ is the potential energy (stiffness) matrix
- g is the force vector
- $\mathbf{c}$ is the vector of expansion coefficients

### Matrix and vector elements:

$$\begin{aligned} \mathrm{T}_{mn} &= \int_V (\phi_i^{(m)})^* \rho \, \phi_i^{(n)} dV & (7) \\ \mathrm{H}_{mn} &= \int_V (\phi_{i,j}^{(m)})^* C_{ijkl} \, \phi_{k,l}^{(n)} dV & (8) \\ \mathrm{g}_m &= \int_V (\phi_i^{(m)})^* \, f_i \, dV. & (9) \end{aligned}$$

### Why “DSM”?

Because we solve Eq. (6) *directly*, rather than by using a normal mode expansion.

## Trial functions for body wave problems

We denote the trial functions in the solid as $\Phi_i^{(\alpha)}$ and $\Phi_i^{(\beta)}$, where $\alpha \rightarrow \{k'\,\ell'\,m'\,p'\}$ and $\beta \rightarrow \{k\,\ell\,m\,p\}$ are respectively pointers to $k'$ and $k$, the indices for linear splines, $\ell'$ and $\ell$, the angular orders, $m'$ and $m$, the azimuthal orders, and $p'$ and $p$, the indices for the polarization of the trial functions.

The depth dependent part of all three types of trial functions is given by linear splines, and the horizontal dependence is given by the three fully normalized vector spherical harmonics, as shown below. The first two types of trial function, $\boldsymbol{\Phi}^{(k\ell m1)}$ and $\boldsymbol{\Phi}^{(k\ell m2)}$, have spheroidal polarization, and the third type, $\boldsymbol{\Phi}^{(k\ell m3)}$, has toroidal polarization. The explicit form of the trial functions in terms of their $(r, \theta, \phi)$ components is

$$
\begin{aligned}
\boldsymbol{\Phi}^{(k\ell m1)} &= X_k(r)\left(Y_{\ell m}(\theta,\phi)\ ,\ 0\ ,\ 0\right) \\
\boldsymbol{\Phi}^{(k\ell m2)} &= X_k(r)\left(0\ ,\ \frac{1}{L}\frac{\partial Y_{\ell m}(\theta,\phi)}{\partial\theta}\ ,\ \frac{1}{L\sin\theta}\frac{\partial Y_{\ell m}(\theta,\phi)}{\partial\phi}\right) \\
\boldsymbol{\Phi}^{(k\ell m3)} &= X_k(r)\left(0\ ,\ \frac{1}{L\sin\theta}\frac{\partial Y_{\ell m}(\theta,\phi)}{\partial\phi}\ ,\ -\frac{1}{L}\frac{\partial Y_{\ell m}(\theta,\phi)}{\partial\theta}\right)
\end{aligned}
$$

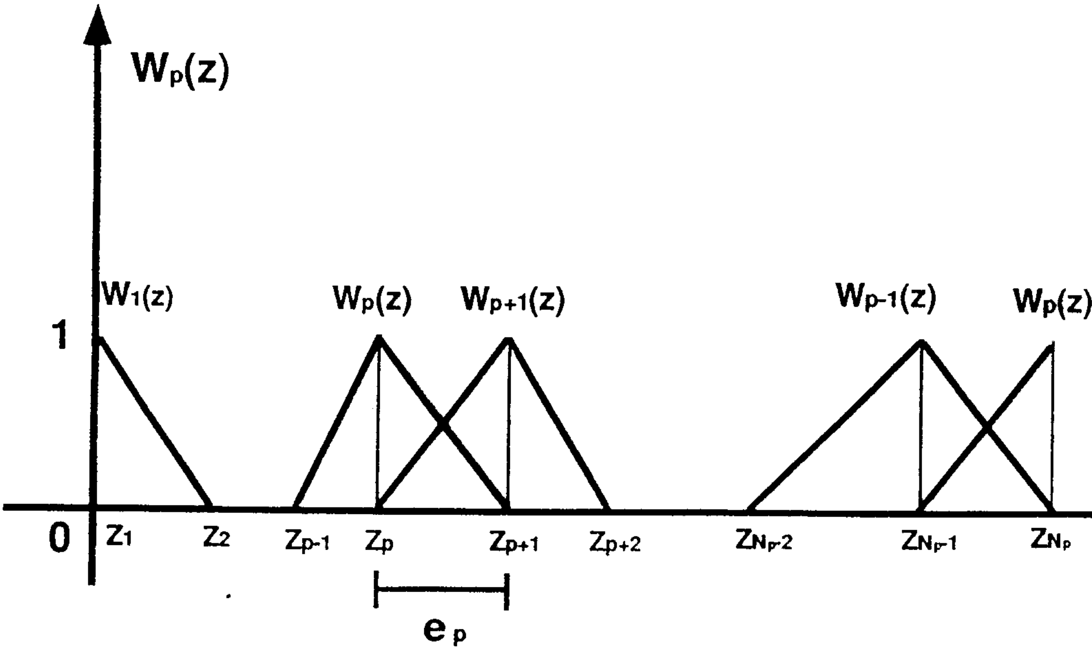
where the $Y_{\ell m}$ are fully normalized surface spherical harmonics and $L = \sqrt{\ell(\ell+1)}$.

## Trial functions for surface wave problems

The eigenfunctions of the degenerate spheroidal and toroidal multiplets of the laterally homogeneous part of the model are used as the trial functions. The first type of trial function, $\boldsymbol{\Phi}^{(k\ell m1)}$ has spheroidal polarization, and the second type, $\boldsymbol{\Phi}^{(k\ell m2)}$, has toroidal polarization:

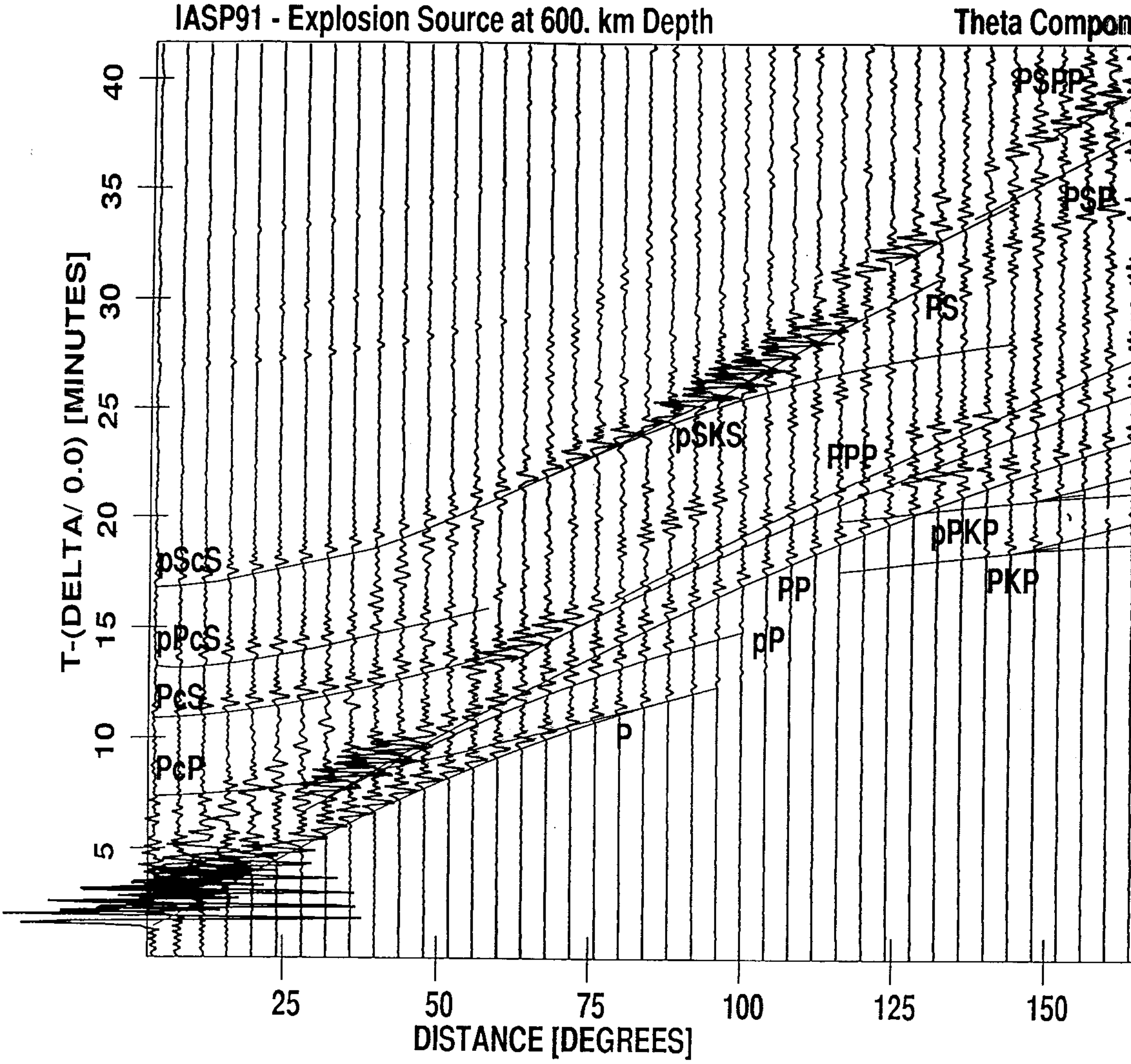
$$
\begin{aligned}
\boldsymbol{\Phi}^{(k\ell m1)} &= U_k(r)\left(Y_{\ell m}(\theta,\phi)\ ,\ 0\ ,\ 0\right) \\
&\quad + V_k(r)\left(0\ ,\ \frac{1}{L}\frac{\partial Y_{\ell m}(\theta,\phi)}{\partial\theta}\ ,\ \frac{1}{L\sin\theta}\frac{\partial Y_{\ell m}(\theta,\phi)}{\partial\phi}\right) \\
\boldsymbol{\Phi}^{(k\ell m2)} &= W_k(r)\left(0\ ,\ \frac{1}{L\sin\theta}\frac{\partial Y_{\ell m}(\theta,\phi)}{\partial\phi}\ ,\ -\frac{1}{L}\frac{\partial Y_{\ell m}(\theta,\phi)}{\partial\theta}\right)
\end{aligned}
$$

**Linear spline trial functions $W_p($**



$W_p(z)$
$W_1(z)$
$W_p(z)$
$W_{p+1}(z)$
$W_{p-1}(z)$
$W_p(z)$
1
0
$z_1$
$z_2$
$z_{p-1}$
$z_p$
$z_{p+1}$
$z_{p+2}$
$z_{N_p-2}$
$z_{N_p-1}$
$z_{N_p}$
$e_p$

Fig. 5


IASP91 - Explosion Source at 600. km Depth
Theta Compon
T-(DELTA/ 0.0) [MINUTES]
40
35
30
25
20
15
10
5
PSPP
PSP
PS
pSKS
PPP
pPKP
PKP
PP
pP
P
pScS
pPcS
PcS
PcP
25
50
75
100
125
150
DISTANCE [DEGREES]

# Efficient algorithm for iterative linearized inversion of seismic waveform data

## Goal:

- Efficient and accurate inversion of seismic waveforms for 3-D earth structure

## Basic principles:

- Accurate calculation of synthetic seismograms and partial derivatives
  - Do not treat laterally heterogeneous structure as an "infinitesimal" perturbation to a 1-D model
  - Conduct iterative linearized waveform inversion with respect to a series of laterally heterogeneous models
- Use all information in seismograms
  - Invert waveforms, not intermediate data

# Stochastic Inverse Problem

We solve the damped generalized inverse problem (the "stochasti inverse"):

$$\left(A^*A \;+\; \gamma^2 I\right)(\delta m) \;=\; A^*(\delta d), \qquad (1$$

where

- $\gamma$ is the damping parameter and $I$ is the identity matrix
- $A_{ij}$ is the partial derivative of the $i$-th datum with respect to th $j$-th model parameter
- $(\delta m)$ is the vector of perturbations to the starting model
- $(\delta d)$ is the vector of residuals
- $A^*(\delta d)$ is the gradient (steepest descent) vector
- For simplicity we assume the source parameters are known.
- The $ll'$-th element of $(A^*A)$ in Eq. (1) is:

$$(A^*A)_{ll'} \;=\; \sum_k \sum_p \left(\frac{\partial u_i^{(k)}(\mathbf{r}^{(p)})}{\partial m_l}\right)^* \left(\frac{\partial u_i^{(k)}(\mathbf{r}^{(p)})}{\partial m_{l'}}\right). \qquad (2$$

- where

$$\left(\frac{\partial u_i^{(k)}(\mathbf{r}^{(p)})}{\partial m_l}\right)$$

  is the partial derivative of the seismogram of the $k$-th earthquak at the $p$-th station with respect to the $l$-th model parameter.

# DIRECT SOLUTION METHOD

Displacement for the $k$-th earthquake expanded in terms of trial func
tions:

$$u_i^{(k)}(\mathbf{r}) \;=\; \sum_{n=1}^{N} c_n^{(k)} \phi_i^{(n)}(\mathbf{r}), \qquad (3$$

## Equation of Motion:

$$\left(\omega^2 \mathbf{T} - \mathbf{H}\right) \mathbf{c}^{(k)} = -\mathbf{g}^{(k)}, \qquad (4$$

- $c_n^{(k)}$ is the expansion coefficient of the $n$-th trial function for th
  $k$-th earthquake
- $\mathbf{g}^{(k)}$ is the force vector for the $k$-th earthquake

## Model parameters

- $\delta\rho(\mathbf{r}) = \Sigma\ \delta m_l\ \rho^{(l)}(\mathbf{r})$
- $\delta C_{ijpq}(\mathbf{r}) = \Sigma\ \delta m_l\ C_{ijpq}^{(l)}(\mathbf{r})$
- $\rho^{(l)}(\mathbf{r})$ and $C_{ijpq}^{(l)}(\mathbf{r})$ are the bases for the density and elastic mod
  uli respectively
- $\delta m_l$ are the unknowns to be determined by the inversion

## Matrix elements

The matrix elements corresponding to the $l$-th model paramete are

$$\begin{aligned} \mathrm{T}_{mn}^{(l)} &= \int_V (\phi_i^{(m)})^* \, \rho^{(l)} \, \phi_i^{(n)} dV & (5 \\ \mathrm{H}_{mn}^{(l)} &= \int_V (\phi_{i,j}^{(m)})^* \, C_{ijpq}^{(l)} \, \phi_{p,q}^{(n)} dV. & (6 \\ \mathrm{g}_m^{(k)} &= \int_V (\phi_i^{(m)})^* \, f_i^{(k)} \, dV. & (7 \end{aligned}$$

## Residuals

The residual for the $i$-component of the seismogram for the $k$-t earthquake at the $p$-th station is given by:

$$(\mathrm{Data} - \mathrm{Synthetic})_i^{(kp)} = \delta d_i^{(kp)} = d_i^{(kp)} - \sum_{n=1}^{N} c_n^{(k)} \phi_i^{(n)}(\mathbf{r}^{(p)}), \quad (8$$

where $d_i^{(kp)}$ is (the Fourier transform of) the $i$-component of the ob served seismogram of the $k$-th earthquake at the $p$-th station.

## Partial derivatives of expansion coefficients

If we make perturbations, $\delta m_l$, to the parameters of the eart model, the perturbation to the expansion coefficients for the syntheti seismogram in Eq. (4), $\delta c^{(k)}$, can be found by solving

$$\left[\left(\omega^2\mathbf{T}-\mathbf{H}\right)+\sum_l \delta m_l\,\left(\omega^2\,\mathbf{T}^{(l)}-\,\mathbf{H}^{(l)}\right)\right]\left(\mathbf{c}^{(k)}+\delta c^{(k)}\right)=-\mathbf{g}^{(k)}.$$

(9

We now use the first order Born approximation to find the partia derivatives. For a small change in the expansion coefficient for a singl model parameter, $\delta m_l$, we obtain

$$\left(\omega^2\mathbf{T}-\mathbf{H}\right)\frac{\delta c^{(k)}}{\delta m_l}=-\,\left(\omega^2\,\mathbf{T}^{(l)}-\,\mathbf{H}^{(l)}\right)\mathbf{c}^{(k)}. \qquad (10$$

We formally write the inverse of Eq. (10) in the form

$$\frac{\delta c^{(k)}}{\delta m_l}=-\left(\omega^2\mathbf{T}-\mathbf{H}\right)^{-1}\,\left(\omega^2\,\mathbf{T}^{(l)}-\,\mathbf{H}^{(l)}\right)\mathbf{c}^{(k)}. \qquad (11$$

Using the identity $(A^{-1})^*=(A^*)^{-1}$, we take the conjugate transpos of both sides of Eq. (11) to obtain

$$\left(\frac{\delta c^{(k)}}{\delta m_l}\right)^*=-\,(\mathbf{c}^{(k)})^*\left(\omega^2\,\mathbf{T}^{(l)}-\,\mathbf{H}^{(l)}\right)^*\left(\left(\omega^2\mathbf{T}-\mathbf{H}\right)^*\right)^{-1}. \qquad (12$$

Equation (12) is used below to absorb $\frac{\delta c^{(k)}}{\delta m_l}$ into the calculations s that the need to know its explicit value is eliminated. This is the key point in our derivation.

## Normal equations and weighted residuals

The complex conjugate of the partial derivative of the seismogra of the $k$-th earthquake at the $p$-th station with respect to the $l$-t model parameter is given by:

$$\left(\frac{\partial u_i^{(k)}(\mathbf{r}^{(p)})}{\partial m_l}\right)^* = \sum_{n=1}^{N}\left(\frac{\delta c_n^{(k)}}{\delta m_l}\right)^*\left(\phi_i^{(n)}(\mathbf{r}^{(p)})\right)^* = \left(\frac{\delta c^{(k)}}{\delta m_l}\right)^* \mathbf{y}_i^{(p)} \qquad (13$$

We use the following intermediate quantities in evaluating Eq. (13)

$$\mathbf{y}_i^{(p)} = \begin{pmatrix} (\phi_i^{(1)}(\mathbf{r}^{(p)}))^* \\ (\phi_i^{(2)}(\mathbf{r}^{(p)}))^* \\ \vdots \\ (\phi_i^{(N)}(\mathbf{r}^{(p)}))^* \end{pmatrix} \qquad (14$$

and

$$\mathbf{z}_i^{(p)} = -\left(\left(\omega^2\mathbf{T}-\mathbf{H}\right)^*\right)^{-1}\mathbf{y}_i^{(p)}. \qquad (15$$

$\mathbf{z}_i^{(p)}$ can be obtained as the solution of

$$\left(\omega^2\mathbf{T}-\mathbf{H}\right)^*\mathbf{z}_i^{(p)} = -\mathbf{y}_i^{(p)}. \qquad (16$$

Note that $\mathbf{z}_i^{(p)}$ must be computed only once for each component each station that is used in the inversion, regardless of the number earthquakes.

## Normal equations and weighted residuals — 2

We repeat Eqs. (12) and (15):

$$\left(\frac{\delta c^{(k)}}{\delta m_l}\right)^* = -(\mathbf{c}^{(k)})^* \left(\omega^2\,\mathbf{T}^{(l)} - \mathbf{H}^{(l)}\right)^* \left(\left(\omega^2\mathbf{T} - \mathbf{H}\right)^*\right)^{-1}$$

$$\mathbf{z}_i^{(p)} = -\left(\left(\omega^2\mathbf{T} - \mathbf{H}\right)^*\right)^{-1} \mathbf{y}_i^{(p)}$$

From Eq. (13) we obtain:

$$\begin{aligned}
\left(\frac{\partial u_i^{(k)}(\mathbf{r}^{(p)})}{\partial m_l}\right)^* &= \left(\frac{\delta c^{(k)}}{\delta m_l}\right)^* \mathbf{y}_i^{(p)} \\
&= -(\mathbf{c}^{(k)})^* \left(\omega^2\,\mathbf{T}^{(l)} - \mathbf{H}^{(l)}\right)^* \left(\left(\omega^2\mathbf{T} - \mathbf{H}\right)^*\right)^{-1} \mathbf{y}_i \\
&= (\mathbf{c}^{(k)})^* \left(\omega^2\,\mathbf{T}^{(l)} - \mathbf{H}^{(l)}\right)^* \mathbf{z}_i^{(p)} \qquad (1
\end{aligned}$$

Equation (17) shows that the partial derivatives of the synthetic seis mograms can be computed without having to know the explicit valu of $\frac{\delta c^{(k)}}{\delta m_l}$.

The $l$-th element of the r.h.s. of the inverse problem Eq. (1) is give by

$$(A^*(\delta d))_l = \sum_k \sum_p \left(\frac{\partial u_i^{(k)}(\mathbf{r}^{(p)})}{\partial m_l}\right)^* \delta d_i^{(kp)}. \qquad (18$$

We now substitute Eq. (17) into Eq. (18) to obtain

$$(A^*(\delta d))_l = \sum_k \sum_p (\mathbf{c}^{(k)})^* \left(\omega^2\,T^{(l)} - H^{(l)}\right)^* \mathbf{z}_i^{(k)}\,\delta d_i^{(kp)} \quad (19$$

# Conclusions

- The inverse problem can be solved without having to know th value of the partial derivatives of the expansion coefficients, $\frac{\delta c^{(k}}{\delta m_l}$ This leads to a tremendous decrease in the volume of computa tions:
  - In both the new and old algorithms we have to solve Eq. (4 once per earthquake to compute the synthetic seismograms
  - In our old algorithm we had to solve Eq. (10) once per earth quake per model parameter — thousands of times, in all.
  - But in our new algorithm, rather than having to solve Eq (10) thousands of times, we now have to solve Eq. (16) onl once per component per station. This allows the CPU tim to be reduced by over a factor of ten, as shown by the table on the next page.
- By reducing the CPU time, the methods presented in this poste allow the number of seismograms and frequency bands to b greatly increased, as compared to our previous work. This mean that in our future work it should be possible to obtain highe resolution information on the earth's 3-D structure.
- Our methods can also be derived for a "pixel" (point-scatterer inversion. A preprint is available on request.

Table 1. Parameters for numerical test

| **Number of:** | **Variable** | **Value** |
|---|---|---|
| Trial Functions | $N$ | 877 |
| Earthquakes | $K$ | 30 |
| Model Parameters | $L$ | 240 |
| Stations | $P$ | 18 |

Table 2. CPU Time required for each operation

| **Operation** | **CPU Time (msec)** |
|---|---|
| LU Factorization | 1015 |
| Back-substitution | 6.35 |
| Sparse multiplication (average for all matrices) – Original | 1.75 |
| Sparse multiplication (average for all matrices) – Improved | 0.27 |

Table 3. CPU Times for both algorithms for both sparse multiplication implementations

| **Old Algorithm** | | | | | |
|---|---|---|---|---|---|
| | Operation Count | | | CPU Time (s | |
| Computation | LU Fact-orizations | Backsub-stitutions | Sparse Multi-plications | Original | Impr |
| $(\omega^2\mathbf{T}-\mathbf{H})\,\mathbf{c}^{(k)} = -\mathbf{g}^{(k)}$ | 1 | $K = 30$ | 0 | 1.206 | |
| $(\omega^2\mathbf{T}-\mathbf{H})\,\frac{\delta c^{(k)}}{\delta m_l} = -\left(\omega^2 \boldsymbol{T}^{(l)} - \boldsymbol{H}^{(l)}\right)\mathbf{c}^{(k)}$ | 0 | $K \times L = 7200$ | $K \times L = 7200$ | 58.320 | 47 |
| TOTAL | 1 | $K \times (L+1) = 7230$ | $K \times L = 7200$ | 59.526 | 48 |
| **New Algorithm** | | | | | |
| | Operation Count | | | CPU Time (s | |
| Computation | LU Fact-orizations | Backsub-stitutions | Sparse Multi-plications | Original | Impr |
| $(\omega^2\mathbf{T}-\mathbf{H})\,\mathbf{c}^{(k)} = -\mathbf{g}^{(k)}$ | 1 | $K = 30$ | 0 | 1.206 | |
| $(\omega^2\mathbf{T}-\mathbf{H})^*\,\mathbf{z}_i^{(p)} = -\mathbf{y}_i^{(p)}$ | 1 | $P = 18$ | 0 | 1.129 | |
| $(\mathbf{c}^{(k)})^*\left(\omega^2 \boldsymbol{T}^{(l)} - \boldsymbol{H}^{(l)}\right)^*$ | 0 | 0 | $K \times L = 7200$ | 12.600 | |
| TOTAL | 2 | $K + P = 48$ | $K \times L = 7200$ | 14.935 | 4 |