Lectures 4 & 5: Interaction of convection and large scales, and tropical dynamics

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Overview

1 Large-scale heat and moisture budgets

- Ascending and descending motion, rainy and dry weather, radiative-convective equilibrium
- Tropical wave dynamics
 - Free wave solutions
- 5 Steady, forced-dissipative (Gill) problem
- Interactive convection quasi-equilibrium theory
 - 7 The moisture mode
- 8 More on convective parameterization
- Onserved variable budgets, gross moist stability
- Observed relationships between moisture and other fields

Following Yanai et al., we write the prognostic equations for dry static energy and specific humidity in flux form in pressure coordinates:

$$\frac{\partial s}{\partial t} + \nabla \cdot s\mathbf{v} + \frac{\partial}{\partial p}(\omega s) = Q_R, \qquad (1)$$

$$\frac{\partial q}{\partial t} + \nabla \cdot q\mathbf{v} + \frac{\partial}{\partial p}(\omega q) = e - c, \qquad (2)$$

Here the dry static energy $s = c_p T + gz$, q is specific humidity, c and e are the condensation and evaporation rates of water (in units of $kg \ m^{-3} \ s^{-1}$), and Q_R is the radiative heating rate ($J \ kg^{-1} \ s^{-1}$, $K \ s^{-1}$ if you divide by c_p). The divergence operator $\nabla \cdot$ represents a horizontal divergence only.

Now consider ensemble averages — or "grid box averages", under the common assumption that **on the scales we are interested in there are many clouds in a "grid box"** — and deviations from those averages, obeying standard Reynolds averaging rules

$$a = \overline{a} + a'$$

 $\overline{ab} = \overline{a}\overline{b} + \overline{a'b'}$

Then apply the averaging operator to the equations, and rearrange:

$$\frac{\partial s}{\partial t} + \nabla \cdot \overline{sv} + \frac{\partial}{\partial p} (\overline{\omega} \ \overline{s}) = Q_1, \qquad (4)$$

$$\frac{\partial q}{\partial t} + \nabla \cdot \overline{q \mathbf{v}} + \frac{\partial}{\partial p} (\overline{\omega} \ \overline{q}) = -Q_2 / I_{\nu}, \tag{5}$$

where

$$Q_{1} = Q_{R} + l_{v}(c - e) - \frac{\partial}{\partial p} \overline{\omega' s'},$$

$$Q_{2} = l_{v}(c - e) + l_{v} \frac{\partial}{\partial p} \overline{\omega' q'}.$$
(6)
(7)

(3)

For convenience we also define

$$Q_{c} = Q_{1} - Q_{R} = I_{v}(c - e) - \frac{\partial}{\partial p} \overline{\omega' s'}$$
(8)

as the "convective heating". Neglecting $\nabla \cdot \overline{s'}\mathbf{v}'$ and $\nabla \cdot \overline{q'}\mathbf{v}'$, (4) and (5) become

$$\frac{\partial s}{\partial t} + \nabla \cdot \overline{s} \, \overline{\mathbf{v}} + \frac{\partial}{\partial p} (\overline{\omega} \, \overline{s}) = Q_1, \tag{9}$$

$$\frac{\partial q}{\partial t} + \nabla \cdot \overline{q} \, \overline{\mathbf{v}} + \frac{\partial}{\partial p} (\overline{\omega} \, \overline{q}) = -Q_2/l_{\nu}, \tag{10}$$

the difference being that the average is applied separately to \mathbf{v} , s, and q on the LHS rather than to quadratic products $\mathbf{v}s$, $\mathbf{v}q$.

Expand the terms on the LHS using the chain rule and then use the form of mass continuity that is valid in pressure coordinates under the hydrostatic approximation,

$$\nabla \cdot \mathbf{v} + \frac{\partial \omega}{\partial p} = 0, \tag{11}$$

to obtain the advective forms

$$\frac{\partial s}{\partial t} + \overline{\mathbf{v}} \cdot \nabla \overline{s} + \overline{\omega} \frac{\partial \overline{s}}{\partial p} = Q_1 = Q_c + Q_R, \tag{12}$$

$$\frac{\partial q}{\partial t} + \overline{\mathbf{v}} \cdot \nabla \overline{q} + \overline{\omega} \frac{\partial \overline{q}}{\partial p} = -Q_2/l_{\nu}, \qquad (13)$$

 Q_c and Q_2 are the effective large-scale heat source and large-scale moisture sink due to convection.

A closed theory, or numerical model, must represent Q_c and Q_2 as functions of resolved variables. This is known as **parameterization**.

The simplest parameterization is **convective adjustment**: convection maintains a moist adiabatic profile, while satisfying all conservation laws in the vertical integral. But there are many others, much more complex...

What profiles of MSE, saturation MSE, Q_1 , Q_2 look like in deep convective regions of the tropics. (Yanai et al. 1973)



Fig. 9. The mean dry static energy \bar{s} , moist static energy \bar{h} (both solid), and saturation moist static energy \bar{h}^* (dashed) of the environment.



FIG. 10. The mean apparent heat source Q_1 (solid) and moisture sink Q_2 (dashed). On the left is the radiational heating rate given by Dopplick (1970).

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Note Q_1 peaks higher than Q_2 ; most condensation occurs lower, where q is higher; turbulence transports condensation-warmed air upwards.

If we integrate the moisture equation in the vertical over the depth of the convective layer (whose boundaries in pressure are p_0 , p_T), use mass conservation, assume **steady state**, and assume that $\overline{\omega q} = 0$ at p_0 and p_T , we get

$$g^{-1} \int_{\rho_T}^{\rho_0} Q_2 \, dp = L_\nu(P - E) = -g^{-1} L_\nu \int_{\rho_T}^{\rho_0} \nabla \cdot (\mathbf{v}q) \, dp, \qquad (14)$$

where $E = -g^{-1}\overline{\omega'q'}|_{p_0} \approx \rho_0 \overline{w'q'}|_{z=0}$ is the surface water vapor flux (evaporation). Similarly,

$$\frac{1}{g}\int_{p_T}^{p_0}Q_cdp=L_vP+H$$

where H is the sensible heat flux (usually $H \ll E$ over ocean).

A comment on tropical atmospheric dynamics:

We can choose to describe the problem of understanding the tropical atmospheric circulation — both weather and climate — as being composed of two parts:

- **(**) Given large-scale fields, find Q_1 and Q_2 ,
- **②** Use Q_1 and Q_2 and the governing equations to evolve the large-scale fields in time.

Part of this problem is finding the large-scale dynamical response of the dry variables — winds, temperatures, pressures — to Q_1 . I claim that **this part is relatively straightforward** and well-understood at this point. It is classic geophysical fluid dynamics.

Finding Q_1 and Q_2 , and understanding how they depend on the large scales, remains the greater challenge.

In the discussion that follows we make the further assumptions:

- The atmosphere is in statistically steady state, so we can neglect the time derivative terms

With these approximations our equations become

$$\overline{\omega}\frac{\partial\overline{s}}{\partial\rho} = Q_1 = Q_c + Q_R, \qquad (15)$$
$$\overline{\omega}\frac{\partial\overline{q}}{\partial\rho} = -Q_2/l_v, \qquad (16)$$

$$\overline{\omega}\frac{\partial\overline{s}}{\partial\rho} = Q_1 = Q_c + Q_R, \overline{\omega}\frac{\partial\overline{q}}{\partial\rho} = -Q_2/l_v, \qquad (17)$$

Keep in mind that as Q_R is generally negative in the troposphere (with a value on the order of 1 K d^{-1} , while Q_c and Q_2 are generally either positive or zero.

Now we consider the different kinds of balances that can occur in these equations.

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We define a state of radiative-convective equilibrium (RCE) as one in which $\omega = 0$, as appropriate for, say, an average over the entire planet. In that case (15) implies

$$Q_c = -Q_R, \tag{18}$$

convective heating balances radiative cooling. Eq. (16) implies

$$Q_2 = 0.$$
 (19)

This does not mean that precipitation is zero. Rather, we can easily show that in RCE,

$$P = E$$
,

precipitation equals surface evaporation, and also

$$I_{v}P + H = -g^{-1}\int_{p_{T}}^{p_{0}}Q_{R} dp,$$

which says that the total condensation heating plus surface sensible heat flux (H) equals the radiative cooling. Going back to the definition of Q_2 , we conclude that the condensation is not itself zero (unless surface evaporation is, which is unlikely over most surfaces, and certainly over ocean) but that it is balanced by upward turbulent moisture transport,

$$e-c=rac{\partial}{\partial p}\overline{\omega'q'}.$$
 (20)

Moisture is evaporated at the surface, transported upward by turbulence (which generally means, in clouds, above the shallow subcloud layer adjacent to the surface), and condenses there. This condensation would tend to increase dry static energy due to the enthalpy of vaporization, but radiative cooling compensates, preventing this warming from occurring.

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There is no large-scale vertical motion, and thus by the mass conservation equation (11), no divergent horizontal velocity. Thus there is no large-scale circulation whatsoever (apart from the somewhat artificial possibility of a purely rotational, nondivergent one which would, under our approximations, transport no energy or moisture).

Now let's consider a region in which there is no deep convection, thus no significant condensation of water in the free troposphere.

We then expect that turbulence will be limited to a shallow boundary layer near the surface, and it is only in this layer that the turbulent transport terms in (6) and (10) are significantly different from zero. Thus in the free troposphere, as we have already assumed c = e = 0, we have $Q_c = Q_2 = 0$ as well.

Assuming that Q_R is not zero, but rather is negative, there is no way we can have a steady balance in the temperature equation unless we have $\omega \partial s / \partial p < 0$ as well. As stable stratification to dry motions implies (and we observe) $\partial s / \partial p < 0$, this implies $\omega > 0$, descent.

The balance is (overbars understood; everything that follows is in terms of averaged variables unless primes are explicitly written):

$$\omega \frac{\partial s}{\partial p} = Q_R. \tag{21}$$

In the moisture budget, we must then have

$$\omega \frac{\partial q}{\partial p} = 0 \tag{22}$$

in the free troposphere. Since we have already found that ω is not zero this implies that

$$\frac{\partial q}{\partial p} = 0, \tag{23}$$

which means q is constant with respect to pressure.

But up near the tropopause the temperature becomes very small, thus by Clausius-Clapeyron so does the saturation vapor pressure and presumably the vapor pressure itself. We can consider the value of q to be set up there, say $q = q_{tp}$, but compared to the saturation values lower down this is not much different from $q \approx 0$.

Thus we expect a **very dry free troposphere** in these descending regions. In reality the degree of dryness will often be somewhat reduced by horizontal advection from moister regions, which we have neglected. In the boundary layer we have

$$Q_2 = l_v \frac{\partial}{\partial p} \overline{\omega' q'},$$

thus our moisture budget is

$$\omega \frac{\partial q}{\partial p} = -\frac{\partial}{\partial p} \overline{\omega' q'},\tag{24}$$

which says that the turbulent transport of moisture up from the surface balances large-scale advective drying.

The dry static energy budget in the boundary layer is

$$\omega \frac{\partial s}{\partial p} = -\frac{\partial}{\partial p} \overline{\omega' s'} + Q_R, \qquad (25)$$

or putting terms which will be of similar sign together,

$$Q_R = \frac{\partial}{\partial \rho} \overline{\omega' s'} + \omega \frac{\partial s}{\partial \rho}, \qquad (26)$$

which says that the turbulent sensible heat transport (which if we get more specific will include a contribution connected directly to the surface sensible heat flux as well as a contribution due to turbulent entrainment of potentially warm free tropospheric air) plus large-scale advective (adiabatic) warming together balance radiative cooling. Now we consider a region in which $Q_c + Q_R > 0$. This implies that $\omega < 0$, as our temperature equation (15) can be written

$$\omega = \frac{Q_c + Q_R}{\partial s / \partial p},.$$
(27)

The vertical integral of Q_2 is constrained once we know Q_c and the surface fluxes H and E, as we can easily derive

$$\int_{p_T}^{p_0} Q_2 \, dp = \int_{p_T}^{p_0} Q_c \, dp - H - I_v E.$$

The vertical structure of Q_2 is not so clearly constrained, but if we know the profiles of q and ω we can infer what it must be from the moisture budget (16).

Further, from (14) we can deduce that in this regime P > E. If $\omega < 0$ throughout the troposphere (as is generally true in deep convective regimes), and furthermore if we assume that the convective layer is bounded in the vertical by points at which $\omega = 0$, it follows that $\partial \omega / \partial p$ is positive in the lower troposphere and negative in the upper.

From that, the fact that q invariably decreases with height (increases with pressure), and the neglect of horizontal gradients, it follows that P > E in this regime, implying net import of water vapor, or "moisture convergence".

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Note that in this regime we expect the turbulent transport terms to be important throughout the convecting layer, which will be the entire troposphere or a large fraction of it. So the distinction between the boundary layer and the free troposphere is less clear here, though often there is still a thin layer well-mixed in s and q just near the surface which we might consider to be the boundary layer.

The weak temperature gradient approximation and the shallow-to-deep cumulus transition

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Climatology: Free tropospheric temperature is homogeneous. SST and precipitation have much more structure, and resemble each other.







At first order, we have a monotonic relationship between SST and precipitation.



Waliser et al. 1993



The monotonic relationship between SST and precipitation will be explained using two principles:

- 1. Tropospheric temperature is horizontally uniform, and
- 2. Deep convection is a response to instability.

Why is the free tropospheric temperature so uniform?



In geostrophic adjustment in a compressible fluid, a localized temperature anomaly will spread out until it reaches the deformation radius.



At the equator, the effective deformation radius is \sim 1500 km in latitude, and the entire circumference of the earth in longitude. That's why temperature is uniform in the tropics.

The equation for potential temperature is

 $\partial_t \theta + u_h \cdot \nabla_h \theta + wS = Q_c + Q_R$,

with Q_c convective heating, Q_R radiative heating, S static stability (proportional to $\partial_z \theta$)

Since θ is held nearly constant (at const. z) by large-scale adjustment, the dominant balance is

 $wS\approx Q_{c}^{}+Q_{R}^{}$

Let's build our understanding of tropical dynamics starting from a single column

$$\partial_t T + wS = Q_c + Q_R,$$

 $\partial_t q + w \partial_z q = Q_q$

with S $\propto \partial_z \theta$, Q_q=(e-c) + ∂_z w' q'

where we have neglected horizontal advection terms $u_h \cdot \nabla_h$ (θ ,q).

A *single column model* is one which solves these equations, with no horizontal dimension.

The convective heating & moistening are parameterized as functionals of T(z), q(z), the temperature and moisture profiles, generally minimizing some measure of conditional instability.

We divide z into a boundary layer (b) and free troposphere (f). Write

 $\begin{aligned} & \mathbf{Q}_{c} \ (\mathsf{T},\mathsf{q}) = \mathbf{Q}_{c}(\mathsf{T}_{b},\mathsf{q}_{b},\mathsf{q}_{f},\mathsf{T}_{f}) \\ & \mathbf{Q}_{q} \ (\mathsf{T},\mathsf{q}) = \mathbf{Q}_{q}(\mathsf{T}_{b},\mathsf{q}_{b},\mathsf{q}_{f},\mathsf{T}_{f}) \end{aligned}$

 T_b and q_b are controlled by the boundary conditions (e.g., SST). T_f must be the same everywhere. For an area small compared to the entire tropics, T_f is an external parameter, not part of the solution. It is determined by convective adjustment to the boundary conditions *elsewhere in the tropics* - to the tropical mean SST, under the simplest assumption.



$$\partial_{t}T + wS = Q_{c}(T_{b},q_{b},q_{f},T_{f}) + Q_{R}(T_{b},q_{b},q_{f},T_{f}),$$

$$\partial_{t}q + w\partial_{z}q = Q_{q}(T_{b},q_{b},q_{f},T_{f})$$

If instead we model the whole tropics (or whole planet), we get radiative-convective equilibrium, w=0. It would be the observed state if the whole globe had uniform SST. In that case T_f must be part of the solution.

The temperature profile adjusts to neutrality to buoyant ascent given T_b , q_b , which depend on the SST.



Manabe and Strickler 1964, *J. Atmos. Sci.* 21, 361-385.

Fig. 3.16 Calculated temperature profiles for radiative equilibrium, and thermal equilibrium with lapse rates of 9.8°C km⁻¹ and 6.5°C km⁻¹. [From Manabe and Strickler (1964). Reprinted with permission from the American Meteorological Society.]

RCE temperature as a function of latitude and pressure, calculated with the Emanuel single column model. Uniform CO2, ozone, surface wind speed.



In RCE, P=E, and atmos. temperature increases ~linearly with SST (slope>1)



In the real tropical atmosphere, free tropospheric temperature doesn't adjust to the local SST. Precipitation does.



180

6ÔE

120E

120W

6ÓW


This is a shallow-to-deep convection transition as air moves over increasing SST!





Stevens (2005, Ann. Rev. Earth. Planet. Sci.)



_____ SST increasing along low-level flow

In the tropics, the dominant balance in the T equation in the free troposphere is not

 Q_{c} + $Q_{R} \approx$ 0, as in RCE but instead

 $\mathbf{Q_{c}}\textbf{+}\mathbf{Q_{R}}\approx\textbf{wS,}$

with T_f itself held nearly constant at const. z by geostrophic adjustment.

RCE may be a useful theoretical construct, but it is not a good approximation to the state of the atmosphere locally. It *does* hold approximately in the tropical mean.

In studying the transition from SCu to Cu, a constant vertical motion profile was assumed. This was ok because both types of shallow cloud layers occur under essentially the same overlying free tropospheric state, dry and warm with large-scale descent.

This will **not work** for the shallow to deep transition, because in that case The large-scale vertical motion must change sign during the transition (if Substantial precipitation rates are to be achieved, representative of the real deep convective regions) We use the WTG system:

$$\begin{split} \partial_{t} \mathsf{T}_{f} &= 0, \\ \mathbf{w}_{f} S = \mathsf{Q}_{c}(\mathsf{T}_{b}, \mathsf{q}_{b}, \mathsf{q}_{f}; \mathsf{T}_{f}) + \mathsf{Q}_{R}(\mathsf{T}_{b}, \mathsf{q}_{b}, \mathsf{q}_{f}; \mathsf{T}_{f}), \\ \partial_{t} \mathsf{T}_{b} + \mathsf{w}_{b} S &= \mathsf{Q}_{c}(\mathsf{T}_{b}, \mathsf{q}_{b}, \mathsf{q}_{f}; \mathsf{T}_{f}) + \mathsf{Q}_{R}(\mathsf{T}_{b}, \mathsf{q}_{b}, \mathsf{q}_{f}; \mathsf{T}_{f}, \mathsf{T}_{s}), \\ \partial_{t} \mathsf{q}_{f} + \mathsf{w} \partial_{z} \mathsf{q}_{f} &= \mathsf{Q}_{q}(\mathsf{T}_{b}, \mathsf{q}_{b}, \mathsf{q}_{f}; \mathsf{T}_{f}) \\ \partial_{t} \mathsf{q}_{b} + \mathsf{w} \partial_{z} \mathsf{q}_{b} &= \mathsf{Q}_{q}(\mathsf{T}_{b}, \mathsf{q}_{b}, \mathsf{q}_{f}; \mathsf{T}_{f}, \mathsf{T}_{s}), \end{split}$$

solution variables (fns of z,t) externally specified; $T_s = SST$

For many purposes it is ok to assume constant relative humidity in boundary layer, and surface air temperature diagnostically related (and close to) surface temperature, thus $q_b \approx q_b(T_b) \approx q_b(T_b(T_s))$

WTG/RCE Calculations with Emanuel single-column models (Renno et al. 1994, *JGR*, **99**, 14429-14441; Bony and Emanuel)

- Emanuel convective parameterization
- Bony-Emanuel cloud scheme (in some calculations)
- Goddard/French radiative schemes
- Fixed SST
- CO2, Ozone at reasonable present-day values
- Surface wind speed=7 m/s
- No horizontal advection of moisture (for WTG, means that horizontal moisture gradient is assumed to vanish)

Precipitation in WTG simulations with Emanuel's model. Dots are from observed January climatology of SST and Precip over oceans 20S-20N.



Sobel and Bretherton 2000

Deep convection is controlled by stability of the sounding (e.g., Arakawa and Schubert 1974). The stability here is determined by the free tropospheric temperature and the SST.



Relative humidity and pressure vertical velocity (ω) profiles as functions of SST in the same set of simulations



The tropospheric temperature is horizontally uniform throughout the tropics. Its value is set by a tropical mean adjustment to the mean SST in deep convective regions.

Near-surface air adjusts to the local surface temperature, while free tropospheric air cannot, since it has to be the same as elsewhere.

Over a relatively cold surface, the sounding is stable. Over a relatively warm surface, the sounding is unstable.

This gives us the zero-order relationship between SST and precipitation.

NB the relationship is between *time-mean* CAPE and precip; on sub-monthly time scales, it doesn't work.

Precipitation in RCE and WTG.



Figure 4. Precipitation rate (mm day⁻¹) as a function of SST in RCE and WTG.

2011 Ramsay and Sobel 2010 Similar calculations with cloud-resolving model, in 2D and 3D (Wang and Sobel 2011, *J. Geophys. Res.*)



Although SST controls the transition in these simulations, it is mediated by water vapor (Wang and Sobel 2011, *J. Geophys. Res.*; see also Wang and Sobel 2012)



ENSO Teleconnections





work with John Chiang http://www.rpdc.tas.gov.au/soer/image/377/index.php (Chiang and Sobel 2002, *J. Climate*) 97-98 El Nino. Early event. Large warm SST anomalies in central/east Pacific, weak anomalies elsewhere. Negative precip anomalies in warm pool.





97-98 El Nino. Late event. Tropical Indian and Atlantic warm up. Negative precip anomalies go away.



Apr 1998

Typical El Nino precip anomalies are negative throughout the tropics, except the eastern/central Pacific. This suggests a global-scale explanation.



WARM EPISODE RELATIONSHIPS DECEMBER - FEBRUARY

Ropelewski and Halpert 1987

Interannual tropospheric temperature anomalies are very homogeneous within the tropics,



FIG. 3. Time series of MSU-2 temperatures averaged over subregions of the Tropics in the 20° N to 20° S latitude belt. Top to bottom: 93° to 145°E, which corresponds to Indonesia; 16°W to 45°E, which corresponds to Africa; and 80° to 40°W, which corresponds to South America. The temperature scale is the same for all three time series: one tick mark corresponds to 0.5 K.

Yulaeva and Wallace 1994



FIG. 4. Temporal correlation between MSU-2 temperature anomalies averaged over the tropical belt (the third time series in Fig. 1) and MSU-2 temperature at each grid point. Contour interval 0.1; the 0.9 contour is thickened, negative contours are dashed, and the zero contour is dashed and thickened.



and related to tropical mean SST anomalies

Sobel, Held, and Bretherton 2002

We user our single-column modeling strategy to understand tropical ENSO teleconnections, now with interactive slab ocean



$$\begin{array}{l} \partial_{t}\mathsf{T}_{\mathsf{f}}=0,\\ \mathbf{w}_{\mathsf{f}}S=\mathsf{Q}_{\mathsf{c}}(\mathsf{T}_{\mathsf{b}},\mathsf{q}_{\mathsf{b}},\mathsf{q}_{\mathsf{f}};\mathsf{T}_{\mathsf{f}})+\mathsf{Q}_{\mathsf{R}}(\mathsf{T}_{\mathsf{b}},\mathsf{q}_{\mathsf{b}},\mathsf{q}_{\mathsf{f}};\mathsf{T}_{\mathsf{f}}),\\ \partial_{t}\mathsf{T}_{\mathsf{b}}+\mathsf{w}_{\mathsf{b}}S=\mathsf{Q}_{\mathsf{c}}(\mathsf{T}_{\mathsf{b}},\mathsf{q}_{\mathsf{b}},\mathsf{q}_{\mathsf{f}};\mathsf{T}_{\mathsf{f}})+\mathsf{Q}_{\mathsf{R}}(\mathsf{T}_{\mathsf{b}},\mathsf{q}_{\mathsf{b}},\mathsf{q}_{\mathsf{f}};\mathsf{T}_{\mathsf{f}},\mathsf{T}_{\mathsf{s}}),\\ \partial_{t}\mathsf{q}_{\mathsf{f}}+\mathsf{w}\partial_{z}\mathsf{q}_{\mathsf{f}}=\mathsf{Q}_{\mathsf{q}}(\mathsf{T}_{\mathsf{b}},\mathsf{q}_{\mathsf{b}},\mathsf{q}_{\mathsf{f}};\mathsf{T}_{\mathsf{f}})\\ \partial_{t}\mathsf{q}_{\mathsf{b}}+\mathsf{w}\partial_{z}\mathsf{q}_{\mathsf{b}}=\mathsf{Q}_{\mathsf{q}}(\mathsf{T}_{\mathsf{b}},\mathsf{q}_{\mathsf{b}},\mathsf{q}_{\mathsf{f}};\mathsf{T}_{\mathsf{f}},\mathsf{T}_{\mathsf{s}})\\ C\partial_{\mathsf{t}}\mathsf{T}_{\mathsf{s}}=\mathsf{net}\;\mathsf{surface}\;\mathsf{heat}\;\mathsf{flux}\end{array}$$

In this case, we force the SCM with time-varying tropospheric temperature, representing that imposed on remote regions by SST in the Pacific.



Sea surface temperature

The ocean surface warming is impressed from above by the free troposphere, via the convective adjustment of the PBL to that, and then surface fluxes



The precipitation response results from a disequilibrium: the upper ocean has not yet had a chance to adjust to the tropospheric warming. Adjustment is slower for a deeper mixed layer. Precip response requires ocean "memory".



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