Waves and their role in the general circulation of the atmosphere

• 1 Nonrotating stratified flow: internal gravity waves and vertical momentum transport
• 2 Quasigeostrophic flow: Waves, instability, and momentum transport
• 3 The circulation of the stratosphere and mesosphere
• 4 Stirring and mixing in the stratosphere: Transport time scales and the distribution of trace gases
• 5 Eddies and tropospheric climate
Lecture 1:

Nonrotating stratified flow: internal gravity waves and vertical momentum transport

(i) 2D nonrotating, stratified flow
(ii) Internal gravity waves
(iii) momentum transport
(iv) Internal gravity wave breaking
(i) 2D nonrotating, stratified flow
Log-pressure coordinates for hydrostatic, compressible, flowx

log-pressure coordinates, pseudoheight

\[
z(p) = -H \ln p
\]

hydrostatic balance (appropriate for large scale, low-frequency waves)

\[
\frac{\partial p}{\partial z_g} = -g \rho_g
\]

constant \( H = RT_*/g \), where \( T_* \) is constant reference temperature

\[
\to \quad dz = -H \frac{dp}{p} = gH \frac{\rho}{p} dz_g = \frac{T_*}{T} dz_g \quad \text{ (} \left| \frac{T}{T_*} - 1 \right| < 0.2 \)
\]

potential temperature

\[
\theta = T(p_*/p)^\kappa
\]

where \( p_* = \text{constant (1000hPa)} \) and \( \kappa = R/c_p = 2/7 \)

(specific entropy = \( c_p \ln \theta + \text{constant} \))

\[
\to \quad c_p T = \Pi(p) \theta \quad \text{ where } \Pi(p) = c_p (p/p_*)^\kappa \text{ is the Exner function}
\]
January climatology of $T$

190K < $T$ < 300K
Log-pressure coordinates for hydrostatic, compressible, flow

log-pressure coordinates, pseudoheight

\[ z(p) = -H \ln p \]

hydrostatic balance (appropriate for large scale, low-frequency waves)

\[ \frac{\partial p}{\partial z_g} = -g \rho_g \]

constant \( H = RT_*/g \), where \( T_\ast \) is constant reference temperature

\[ \rightarrow \quad dz = -H \frac{dp}{p} = gH \frac{\rho}{p} dz_g = \frac{T_\ast}{T} dz_g \quad \left( \left| \frac{T}{T_\ast} - 1 \right| < 0.2 \right) \]

potential temperature

\[ \theta = T(p_*/p)^\kappa \]

where \( p_* \) = constant (1000hPa) and \( \kappa = R/c_p = 2/7 \)

(specific entropy = \( c_p \ln \theta + \text{constant} \))

\[ \rightarrow \quad c_p T = \Pi(p) \theta \quad \text{where} \quad \Pi(p) = c_p (p/p_*)^\kappa \text{ is the Exner function} \]
Two-dimensional hydrostatic, compressible, nonrotating flow

(1) momentum
pressure gradient force per unit mass

\[- \frac{1}{\rho_g} \left( \frac{\partial p}{\partial x} \right)_{z_g} = \frac{1}{\rho_g} \left( \frac{\partial p}{\partial z_g} \right) \left( \frac{\partial z_g}{\partial x} \right) = -\frac{\partial \phi}{\partial x} ; \quad \phi = gz_g\]

\[\rightarrow \frac{du}{dt} = -\frac{\partial \phi}{\partial x} + F ; \quad F \text{ is other (e.g. frictional) force per unit mass}\]

(2) mass continuity
mass element is \[\rho_g \, dx \, dy \, dz_g = \frac{p(z)}{gH} \, dx \, dy \, dz\]
so log-\(p\) coordinate density is

\[\rho = \frac{p}{gH} \rightarrow \rho \text{ constant at constant } p\]

\[p(z) = p_0 \exp \left( -\frac{z}{H} \right) \rightarrow \rho(z) = \rho_0 \exp \left( -\frac{z}{H} \right) , \quad \rho_0 = \frac{p_0}{gH}\]

\[\rightarrow \text{mass per unit area between coordinate surfaces } z, z + dz \text{ constant, so mass flux is nondivergent:}\]

\[\nabla \cdot (\rho \mathbf{u}) = 0\]
(3) entropy budget

\[ \rho c_p \frac{dT}{dt} - \frac{dp}{dt} = J \rightarrow \frac{d\theta}{dt} = (\rho \Pi)^{-1} J \]

\((J\) is heating rate per unit volume\)

(4) hydrostatic balance

\[ \frac{\partial z_g}{\partial p} = -\frac{1}{g \rho_g} \]

\[ \frac{\partial \phi}{\partial z} = \frac{g \partial z_g}{-H p^{-1} \partial p} = \frac{g p}{H} \frac{1}{g \rho_g} = \frac{R}{H} T \quad \text{(ideal gas law)} \]

\[ \rightarrow \frac{\partial \phi}{\partial z} = \frac{\kappa \Pi}{H} \theta \]
Two-dimensional hydrostatic, compressible, nonrotating flow

Full set of equations

\[
\begin{align*}
\frac{du}{dt} &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{\partial \phi}{\partial x} + F \\
\frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial (\rho w)}{\partial z} &= 0 \\
\frac{d\theta}{dt} &= \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + w \frac{\partial \theta}{\partial z} = (\rho \Pi)^{-1} J \\
\frac{\partial \phi}{\partial z} - \frac{\kappa}{H} \Pi \theta &= 0
\end{align*}
\]
(ii) Internal gravity waves
2D internal gravity waves in a compressible fluid (simplest case)

inviscid, adiabatic \( (F = 0 = J) \)

motionless basic state

\[
\theta = \theta_0(z)
\]

\[
\phi_0(z) = \kappa H^{-1} \int_0^z \theta_0(z') \Pi(z') \, dz'
\]

small amplitude perturbations \( \varepsilon << 1 \)

[neglect terms \( O(\varepsilon^2) \)]

\[
\frac{\partial u'}{\partial t} + \frac{\partial \phi'}{\partial x} = 0
\]

\[
\frac{\partial u'}{\partial x} + \frac{1}{\rho} \frac{\partial (\rho w')}{\partial z} = 0
\]

\[
\frac{\partial \theta'}{\partial t} + w' \frac{d\theta_0}{dz} = 0
\]

\[
\frac{\partial \phi'}{\partial z} - \frac{\kappa}{H} \Pi \theta' = 0
\]
All coefficients are functions of $z$, look for solutions

$$
\begin{pmatrix}
u' \\
w' \\
\phi' \\
\theta'
\end{pmatrix}
= \text{Re}
\begin{pmatrix}
U(z) \\
W(z) \\
\Phi(z) \\
\Theta(z)
\end{pmatrix}
\exp[i(kx + ly - \omega t)]
$$

All coefficients are functions of $z$, look for solutions

\[
\frac{\partial u'}{\partial t} + \frac{\partial \phi'}{\partial x} = 0
\]
\[
\frac{\partial u'}{\partial x} + \frac{1}{\rho} \frac{\partial (\rho w')}{\partial z} = 0
\]
\[
\frac{\partial \theta'}{\partial t} + w' \frac{d\theta_0}{dz} = 0
\]
\[
\frac{\partial \phi'}{\partial z} - \frac{\kappa}{H} \Pi \theta' = 0
\]

\[
- i\omega U + ik\Phi = 0
\]
\[
- i\omega \Theta + W \frac{d\theta_0}{dz} = 0
\]
\[
\frac{d\Phi}{dz} - \frac{\kappa}{H} \Pi \Theta = 0
\]
Reduce to single equation for $\Phi$:

\[
e^{z/H} \frac{d}{dz} \left( \frac{\omega^2}{N^2} e^{-z/H} \frac{d\Phi}{dz} \right) + (k^2 + l^2) \Phi = 0
\]

where

\[
N^2(z) = \frac{\kappa}{H} \prod \frac{d\theta_0}{dz} = \frac{g}{T_*} \left( \frac{dT_0}{dz} + \frac{\kappa}{H} T_0 \right)
\]

$\rightarrow$ square of buoyancy frequency

Solution for constant $N^2$:

\[
\phi' = \text{Re} \Phi_0 \exp\left(\frac{z}{2H}\right) \exp\{i(kx + mz - \omega t)\}
\]

where

\[
m = \pm \sqrt{\frac{N^2 k^2}{\omega^2} - \frac{1}{4H^2}}
\]

or

\[
\omega = \pm N \sqrt{\frac{k^2}{m^2 + 1/4H^2}}
\]

Note that if $m$ real:
(i) wave propagates in vertical and
(ii) grows with height as \(e^{z/2H} \sim \rho^{-1/2}\)
Assume $m^2 \gg 1/4H^2 \rightarrow 2\pi/m \ll 4\pi H \approx 100\text{km}$
— good assumption for important atmospheric waves

$$\omega = \pm N \frac{k}{m} = \pm N \tan \gamma$$

($\gamma = \tan^{-1} k/m$); nonhydrostatic case: $\omega = \pm N \sin \gamma$

(\text{hydrostatic approximation valid for } \omega \ll N) group velocity:

$$c_g = \left( \frac{\partial \omega}{\partial k}, \frac{\partial \omega}{\partial m} \right) = \pm \frac{N}{m} \left( 1, -\frac{k}{m} \right)$$

(i) $c_g \cdot k = 0$: group propagation is along phase lines, at angles $\pm \gamma$
(ii) continuity eq. $k \cdot u' = 0$ – fluid motions are along phase lines
(\text{iii) vertical components of group and phase velocities have opposite signs.})
From localized source of frequency $\omega$, waves form rays at angles $\gamma = \sin^{-1}(\omega/N)$ to horizontal, with phase propagation across rays:

LINK to MOVIE
Waves in shear
(slowly varying background state, varies on height scale $h \gg m^{-1}$)

\[
\phi' = \text{Re} \Phi(z)e^{ikx} = \text{Re} \Phi_0(z) \exp\left(\frac{z}{2H}\right) \exp[i(kx + mz - \omega t)]
\]

$\Phi(z)$ slowly varying [$m|\Phi_0| \gg |d\Phi_0/dz|$. $m = m(z)$, also slowly varying.

\[
\begin{align*}
- i \omega U + ik \Phi &= 0 \quad \Rightarrow \quad U = \frac{k}{\omega} \Phi = \frac{k}{\omega} \Phi_0 e^{z/2H} e^{imz} \\
iki U + \frac{1}{\rho} \frac{d}{dz}(\rho W) &= 0 \\
- i \omega \Theta + W \frac{d\theta_0}{dz} &= 0 \quad \Rightarrow \quad W = \frac{i \omega}{d\theta_0/dz} \Theta = \frac{i \omega}{N^2} \left( \frac{1}{2H} + im \right) \Phi_0 e^{z/2H} e^{imz}
\end{align*}
\]

\[
\frac{d\Phi}{dz} - \frac{\kappa}{H} \Pi \Theta = 0 \quad \Rightarrow \quad \Theta = \frac{H}{\kappa \Pi} \frac{d\Phi}{dz} = \frac{H}{\kappa \Pi} \left( \frac{1}{2H} + im \right) \Phi_0 e^{z/2H} e^{imz}
\]

\[
\overline{u' w'} = \frac{1}{2} \text{Re}(U W^*) = -\frac{km}{2N^2} |\Phi_0|^2 e^{z/H}
\]
Waves in shear
(slowly varying background state, varies on height scale \( h \gg m^{-1} \))

\[
\phi' = \text{Re} \Phi(z)e^{ikx} = \text{Re} \Phi_0(z) \exp\left(\frac{z}{2H}\right) \exp[i(kx + mz - \omega t)]
\]

\( \Phi(z) \) slowly varying \[ m|\Phi_0| \gg |d\Phi_0/dz| \]. \( m = m(z) \), also slowly varying.

Momentum flux is constant:  (we’ll see this later)

\[
F_0 = \rho u'w' = -\frac{1}{2} \frac{km(z)}{N^2(z)} |\Phi_0(z)|^2
\]

\[
\rightarrow |\Phi_0(z)|^2 = -2 \frac{F_0}{\rho_0 k} \frac{N^2(z)}{m(z)}
\]

so

\[
\phi' = \left( \frac{2F_0}{\rho_0} \right)^\frac{1}{2} \text{Re} \left[ \frac{N^2(z)}{k |m(z)|} \right]^{\frac{1}{2}} \exp\left(\frac{z}{2H}\right) \exp[i(kx + mz - \omega t)]
\]

\[
\text{varying mean state density factor (usually dominates)}
\]
\[
\phi' = \left( \frac{2F_0}{\rho_0} \right)^{\frac{1}{2}} \text{Re} \left[ \frac{N^2(z)}{k |m(z)|} \right]^{\frac{1}{2}} \exp\left( \frac{z}{2H} \right) \exp[i(kx + mz - \omega t)]
\]

\[
c_{g,z} = \pm \frac{km}{N^2} (\bar{u} - c)^3 \approx \frac{k}{N} (c - \bar{u})^2
\]

Typical values:

\[
2\pi/k = 500\text{km}, \quad c - \bar{u} = 30\text{ms}^{-1}, \quad N^2 = 4 \times 10^{-4}\text{s}^{-2}
\]

\[
c_{g,z} \approx 5\text{ms}^{-1}
\]

→ 0 to 100 km in 20000s ≈ 6 hr
→ weakly dissipated
(iii) momentum transport
Zonal Means

Define (Eulerian) zonal mean for \(a(x, y, z, t)\):
[periodic in \(x\): \(a(x + L, y, z, t) = a(x, y, z, t)\)]

\[
\bar{a}(y, z, t) = \frac{1}{L} \int_0^L a(x, y, z, t) \, dx
\]

eddy (wave) component

\[
a'(x, y, z, t) = a(x, y, z, t) - \bar{a}(y, z, t)
\]

by definition

\[
\overline{a'} = 0 \quad ; \quad \left(\frac{\partial a}{\partial x}\right) = 0 \quad ;
\]

\[
\left(\frac{\partial a}{\partial [y, z, t]}\right) = \frac{\partial \bar{a}}{\partial [y, z, t]}
\]

\[
a \frac{\partial b}{\partial x} = \left(\frac{\partial a}{\partial x} \cdot a\right) - b \frac{\partial a}{\partial x} = -b \frac{\partial a}{\partial x}
\]
Action of waves on the mean state

Mean momentum eq.:

\[
\frac{\partial \bar{u}}{\partial t} + \bar{w} \frac{\partial \bar{u}}{\partial z} = \bar{G} - u' \frac{\partial u'}{\partial x} - w' \frac{\partial u'}{\partial z}
\]

\[
= \bar{G} - \frac{1}{\rho} \frac{\partial}{\partial z} (\rho \bar{u}' \bar{w}') + u' \left( \frac{\partial u'}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial z} (\rho w') \right)
\]

\[
= \bar{G} - \frac{1}{\rho} \frac{\partial}{\partial z} (\rho \bar{u}' \bar{w}')
\]

Mean continuity eq.:

\[
\frac{\partial \bar{u}}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial z} (\rho \bar{w}) = \frac{1}{\rho} \frac{\partial}{\partial z} (\rho \bar{w}) = 0
\]

\[
\rightarrow \bar{w} = 0 \text{ everywhere, if zero on } z = 0 \text{ and}
\]

\[
\frac{\partial \bar{u}}{\partial t} = \bar{G} - \frac{1}{\rho} \frac{\partial}{\partial z} (\rho \bar{u}' \bar{w}')
\]

Similarly,

\[
\frac{\partial \bar{\theta}}{\partial t} = (\rho \Pi)^{-1} \bar{J} - \frac{1}{\rho} \frac{\partial}{\partial z} (\rho w' \bar{\theta}')
\]

\[
\rightarrow \text{eddy fluxes of momentum, } \rho \bar{u}' \bar{w}', \text{ and heat } \rho w' \bar{\theta}'.
\]
Eddy fluxes for steady, inviscid, adiabatic waves in shear

linearized equations

\[
\begin{align*}
\frac{\partial u'}{\partial t} + u_0 \frac{\partial u'}{\partial x} + w' \frac{\partial u_0}{\partial z} + \frac{\partial \phi'}{\partial x} &= G' \\
\frac{\partial u'}{\partial x} + \frac{1}{\rho} \frac{\partial (\rho w')}{\partial z} &= 0 \\
\frac{\partial \theta'}{\partial t} + u_0 \frac{\partial \theta'}{\partial x} + w' \frac{\partial \theta_0}{\partial z} &= (\rho \Pi)^{-1} J' \\
\frac{\partial \phi'}{\partial z} - \frac{\kappa}{H} \tilde{\Pi} \theta' &= 0
\end{align*}
\]

(1) eddy heat flux

Multiply 3rd eq. by \( \theta' \) and average:

\[
\overline{\theta' \frac{\partial \theta'}{\partial t} + u_0 \theta' \frac{\partial \theta'}{\partial x} + w' \theta' \frac{\partial \theta_0}{\partial z}} = \overline{\theta' J'}
\]

But \( \overline{\theta' \frac{\partial \theta'}{\partial x}} = \frac{1}{2} \overline{\theta'^2} \frac{\partial^2}{\partial x} = 0 \); if wave amplitudes are steady, \( \overline{\theta'^2} \) is steady in time, for adiabatic eddies \( (J' = 0) \) then,

\[
\overline{w' \theta'} = 0
\]

→ steady, adiabatic \( (J' = 0) \) waves have zero vertical heat flux.
If phase tilt is as shown:

- $u'$, $w'$, positively correlated
- $\rightarrow$ momentum flux $> 0$

$v' < 0 \quad w' < 0$
$v' > 0 \quad w' > 0$
Momentum flux for steady, conservative \((G' = J' = 0)\) waves (detailed derivation)

First take mean of \(u' \times \text{eddy momentum equation:}\)

\[
\overline{u'} \frac{\partial u'}{\partial t} + u_0 \overline{u'} \frac{\partial u'}{\partial x} + \overline{u'w'} \frac{\partial u_0}{\partial z} + u' \overline{\frac{\partial \phi'}{\partial x}} = u' \overline{G'}
\]

\[
\rightarrow \overline{u'w'} \frac{\partial u_0}{\partial z} + u' \frac{\partial \phi'}{\partial x} = 0
\]

for steady \textit{conservative} waves. But

\[
\overline{u'} \frac{\partial \phi'}{\partial x} = \frac{\partial}{\partial x} (u' \phi') - \phi' \frac{\partial u'}{\partial x} = \frac{1}{\rho} \phi' \frac{\partial}{\partial z} (\rho w')
\]

\[
= \frac{1}{\rho} \frac{\partial}{\partial z} (\rho w' \phi') - w' \frac{\partial \phi'}{\partial z}
\]

\[
= \frac{1}{\rho} \frac{\partial}{\partial z} (\rho w' \phi') + \frac{\kappa}{H} \Pi \overline{w' \theta'}
\]

\[
= \frac{1}{\rho} \frac{\partial}{\partial z} (\rho w' \phi')
\]

\[
\rightarrow \rho \overline{u'w'} \frac{\partial u_0}{\partial z} + \frac{\partial}{\partial z} \left( \rho \overline{w' \phi'} \right) = 0
\]

for steady, \textit{conservative} waves.
\[
\frac{\partial u'}{\partial x} + \frac{1}{\rho} \frac{\partial (\rho w')}{\partial z} = 0
\]

From continuity, define streamfunction \( \xi \) such that

\[
w' = -\frac{\partial \xi'}{\partial x} ;
\]
\[
u' = \frac{1}{\rho} \frac{\partial}{\partial z} (\rho \xi') .
\]

Then write momentum eq. (since \( \partial/\partial t = -c \partial/\partial x \))

\[
(\bar{u} - c) \frac{\partial u'}{\partial x} + \frac{du}{dz} \frac{\partial \xi'}{\partial x} = -\frac{\partial \phi'}{\partial x}
\]
\[
\rightarrow (\bar{u} - c)u' + \frac{du}{dz} \xi' = -\phi'
\]

But

\[
\bar{w}' \bar{\xi}' = -\bar{\xi}' \frac{\partial \bar{\xi}'}{\partial x} = 0
\]

so

\[
(\bar{u} - c)\bar{u}' \bar{w}' = -\bar{w}' \phi'
\]

and

\[
(u - c) \frac{\partial}{\partial z} (\rho \bar{u}' \bar{w}') = 0
\]
Summary

steady, adiabatic, inviscid, waves ($\bar{u} \neq c$):

$$\bar{w}'\theta' = 0 \ ; \ \frac{\partial}{\partial z} \left( \rho \bar{u}'w' \right) = 0$$

momentum flux is constant — manifestation of *wave activity* conservation.
[NB: $\partial \left( \rho \bar{w}'\phi' \right) / \partial z \neq 0$, if $\partial \bar{u}/\partial z \neq 0 \rightarrow \text{“energy flux” not constant}]

Forcing of mean state:

$$\frac{\partial \bar{u}}{\partial t} = \bar{G} - \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho \bar{u}'w' \right)$$

$$\frac{\partial \bar{\theta}}{\partial t} = (\rho \Pi)^{-1} \bar{J} - \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho \bar{w}'\theta' \right)$$

special case of the *nonacceleration theorem*:
mean flow is indifferent to the presence of steady, conservative waves (unless waves influence $\bar{G}, \bar{J}$).
Sign of the momentum flux

\[ c_0 = \frac{\omega}{k} = \pm N \left( m^2 + \frac{1}{4H^2} \right)^{-1/2} \]

add mean flow \( \bar{u} \):

\[ c = c_0 + \bar{u} = \bar{u} \pm N \left( m^2 + \frac{1}{4H^2} \right)^{-1/2} \]

\[ c_{gz} = k \frac{\partial c}{\partial m} = \mp Nkm \left( m^2 + \frac{1}{4H^2} \right)^{-3/2} = \mp \frac{km}{N^2} (c - \bar{u})^3 \]

Upward propagating wave: \( c_{gz} > 0 \rightarrow \text{sgn}(km) = \text{sgn}(\bar{u} - c) \).

\[ \phi' = \operatorname{Re} \Phi(z) \exp\left(-\frac{z}{2H}\right) \exp[i(kx + mz - \omega t)] \]

\[ \rightarrow \rho \bar{u} \bar{w}' = -\frac{1}{2} \rho_0 \frac{km}{N^2} |\Phi(z)|^2 \]

\[ \rightarrow \text{sgn}(\rho \bar{u} \bar{w}') = -\text{sgn}(km) = \text{sgn}(c - \bar{u}) \]

→ momentum flux is nonzero for \( m \neq 0 \), and its sign is that of \( c - u \) ("pseudomomentum rule")
(iv) internal gravity wave breaking
Gravity wave breaking (of the simplest kind)  
(Lindzen, JGR, 86, 9707, 1981; JAS, 42, 301, 1985)

Wave breaks by convective instability where

\[
\frac{\partial \theta}{\partial z} = \frac{\partial \tilde{\theta}}{\partial z} + \frac{\partial \theta'}{\partial z} = \frac{\partial \tilde{\theta}}{\partial z} \left[ 1 + \frac{\partial \theta' \partial \overline{\theta}}{\partial \overline{\theta} \partial z} \right] < 0
\]

\[
\phi' = \text{Re} \left( \frac{2F_0}{\rho_0 k} \right)^{\frac{1}{2}} \left[ -\frac{N^2(z)}{m(z)} \right]^{\frac{1}{2}} \exp \left( \frac{z}{2H} \right) \exp \left[ i \left( kx + mz - \omega t \right) \right]
\]

\[
\frac{\partial \phi'}{\partial z} - \frac{\kappa}{H} \Pi \theta' = 0
\]

\[
\frac{\partial \theta'}{\partial z} \approx \text{Re} \left( \frac{2F_0}{\rho_0 k} \right)^{\frac{1}{2}} \frac{HN^{5/2}}{\kappa \Pi (c - \tilde{u})^2 \sqrt{-m(z)}} \exp \left[ \frac{z}{2H} \right] \exp \left[ i \left( kx + mz - \omega t \right) \right]
\]

\[
\left| \frac{\partial \theta' \partial \overline{\theta}}{\partial \overline{\theta} \partial z} \right| = \left( \frac{2F_0}{\rho_0 k} \right)^{\frac{1}{2}} \frac{N}{(c - \tilde{u})^2 \sqrt{-m(z)}} e^{\frac{z}{2H}} \sim \sqrt{\frac{N}{(c - \tilde{u})^3}} e^{\frac{z}{2H}} \quad \text{(for } m^2 \gg \frac{1}{4H^2})
\]

— breaking favored at large \( z \) and/or small \( |c - \tilde{u}| \)

— breaking favored at large \( z \) and/or small \( |c - \tilde{u}| \)
Internal gravity wave breaking can reinforce zonal flow (we’ll see importance of this later)
Oscillating mean flow can be produced by two upward propagating waves of opposite zonal phase speed:
“QBO” in the lab

subcritical forcing
“QBO” in the lab

supercritical forcing
References

- Lindzen, R. S., 1985: Multiple Gravity-Wave Breaking Levels, J. Atmos. Sci., 42, 301-305