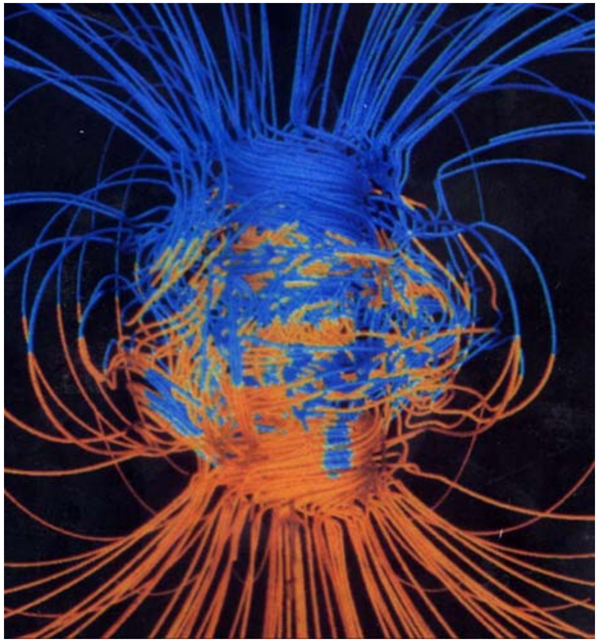
Self-sustained dynamos

The understanding of planetary dynamos has greatly benefited from numerical models. Here we will analyse some model results applying the concepts discussed in lectures 6-9.
The first realistic geodynamo models have been published in 1995 (Glatzmaier & Roberts; Kageyama & Sato), 25 years after the first mantle convection models. Why took it so long?

Cowling's theorem: An axisymmetric field cannot be generated by a homogeneous dynamo (one out of several „anti-dynamo theorems" in kinematic dynamo theory).



The magnetic field must have some asymmetry and must be three-dimensional. Simplified 2D-models are not possible.

In the last years, direct numerical simulations of convection-driven MHD dynamos in rotating spherical shells have become widespread.

MHD dynamo equations

$$\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\vec{\nabla}\vec{u}\right) + 2\vec{e}_z \times \vec{u} + \vec{\nabla}P = E\,\nabla^2\vec{u} + \frac{RaE^2}{\Pr}\frac{\vec{r}}{r_o}T + (\vec{\nabla}\times\vec{B})\times\vec{B}$$

$$\frac{\partial T}{\partial t} + \vec{u}\cdot\vec{\nabla}T = \frac{E}{Pr}\nabla^2 T \qquad \frac{\partial \vec{B}}{\partial t} - \vec{\nabla}\times(\vec{u}\times\vec{B}) = \frac{E}{Pm}\nabla^2\vec{B}$$

The equations are the same as in 9.1 except that a different scaling is used (time scale is Ω^{-1} and magnetic field scale is $(\rho\mu_o)^{1/2}\Omega D$). The Chandrasehkar number disappears as an independent parameter, as no fixed B_o is imposed.

Control parameters

Parameter	Definition	Force balance	Model value	Earth value
Rayleigh number	$Ra=\alpha g DTD^3/\kappa\nu$	buoyancy/diffusion	$\leq 50\times Ra_{crit}$	$5000 x Ra_{crit}$?
Ekman number	$E = \nu / \Omega D^2$	viscosity/Coriolis	$\geq 10^{-6}$	10^{-14}
Prandtl number	$Pr = \nu/\kappa$	viscosity/therm diff.	0.1 – 10	0.1 - 1
magn. Prandtl no.	$Pm = \nu/\lambda$	thermal/magn. diff.	0.06 – 10	10^{-6}

Model values of several parameters are far off from Earth values (viscosity and thermal diffusivity to large and/or rotation too slow.

Model dynamo properties

Output parameters

Parameter	Definition	Force balance	Model value	Earth value
Elsasser number	$\Lambda = B^2/\rho\mu_o\lambda\Omega$	Lorentz / Coriolis	0.1 – 100	1 – 10 ?
Reynolds number	$Re = UD/\nu$	inertia/viscosity	10 – 1000	10^8
Magnet. Reynolds	$Rm = UD/\lambda$	advection/magnetic diff.	40 – 1000	500 - 1500
Rossby number	$Ro = U/\Omega D$	inertia/Coriolis	$10^{-4} – 10^{-1}$	10^{-6}

Elsasser number in correct $\Rightarrow$ Primary force balance Lorentz / Coriolis correct

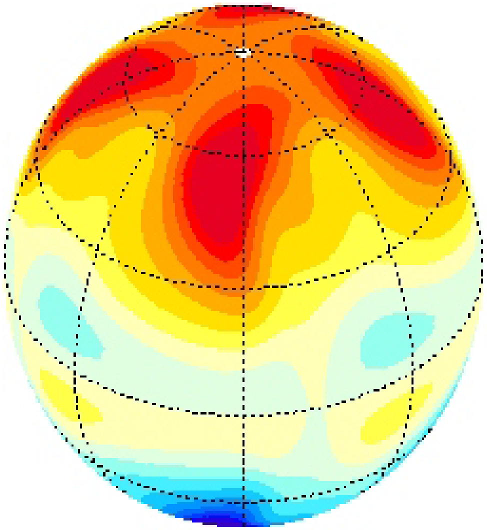
Reynolds number too low $\Rightarrow$ small turbulent flow scales are missed

Magnet. Reynolds number correct $\Rightarrow$ induction process properly modelled

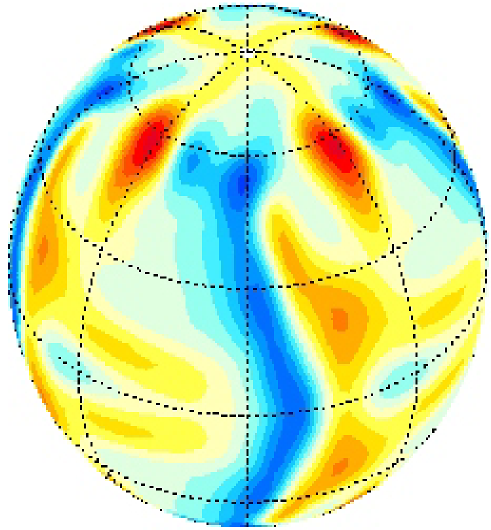
Rossby number too large $\Rightarrow$ inertia over-emphasized

Symbols: U,B – characteristic (e.g. rms) values of velocity and magnetic field, D – thickness of spherical shell

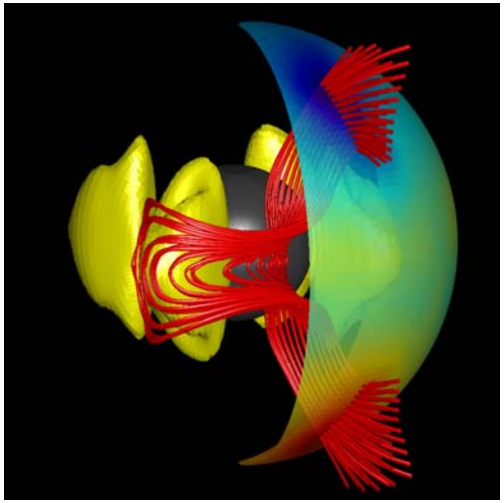
A simple dynamo model



B_r at $r=r_o$



u_r at $r=0.9r_o$



Magnetic field lines

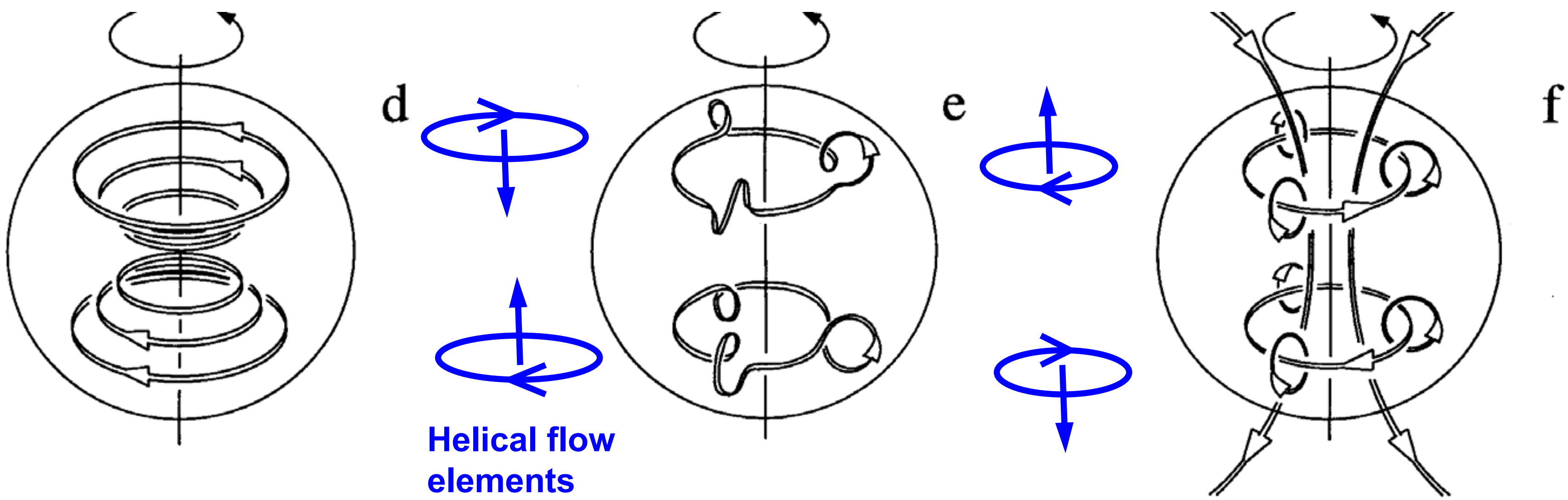
Benchmark dynamo: $Ra=10^5$, $E=10^{-3}$, $Pr=1$, $Pm=5$. One of the simplest dynamo models ever found. The solution is stationary, aside from a drift in φ. It has fourfold symmetry in φ. The external magnetic field is strongly dipolar, with the magnetic flux concentrated in four flux lobes. $Rm=39$ (near minimum required for a dynamo), $\Lambda \approx 3$. The simplicity of the structures allows to study the magnetic field generation mechanism.

Dynamo mechanisms

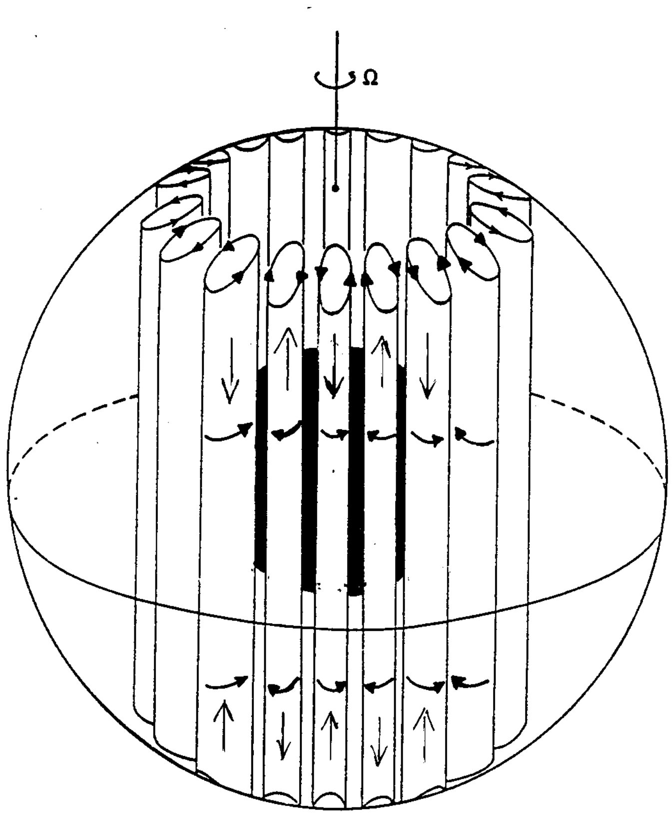
The term „field generation mechanism“ can be understood differently:

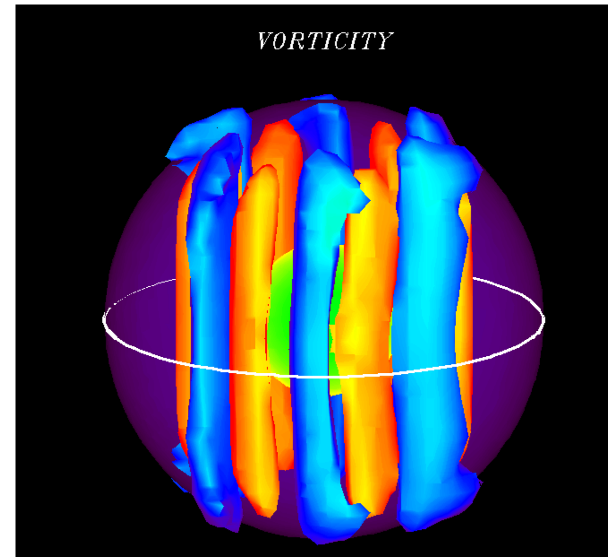
1) The way kinetic energy is converted to magnetic energy $\Rightarrow$ field line stretching (8.5).
2) The way how poloidal magnetic field is converted to toroidal field (in particular the axisymmetric components of both fields). Neither a purely poloidal nor a purely toroidal field can be generated by a homogeneous dynamo („anti-dynamo theorem“).

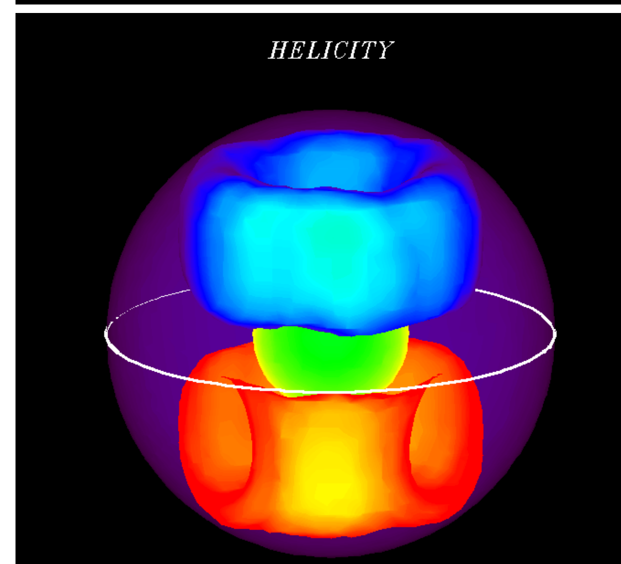
We will consider the second question. The generation of an axisymmetric toroidal field from a poloidal (dipole) field by the ω-effect (shear by differential rotation) has been illustrated in 8.5. For the generation of a dipole field from a toroidal field, small-scale helical motion plays an important role. In a frozen flux approximation, they create small poloidal loops of magnetic field that can link up by magnetic diffusion to build a large-scale dipole field.



Helical flow in rotating convection

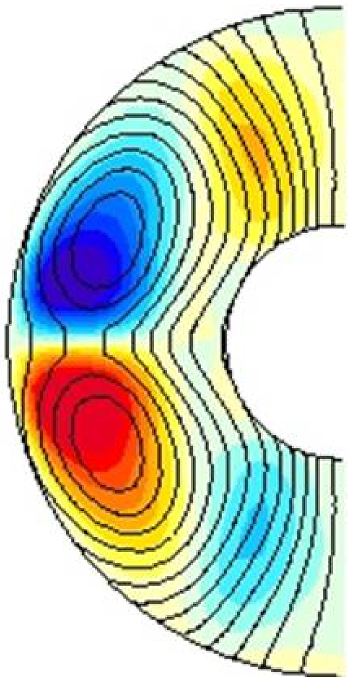




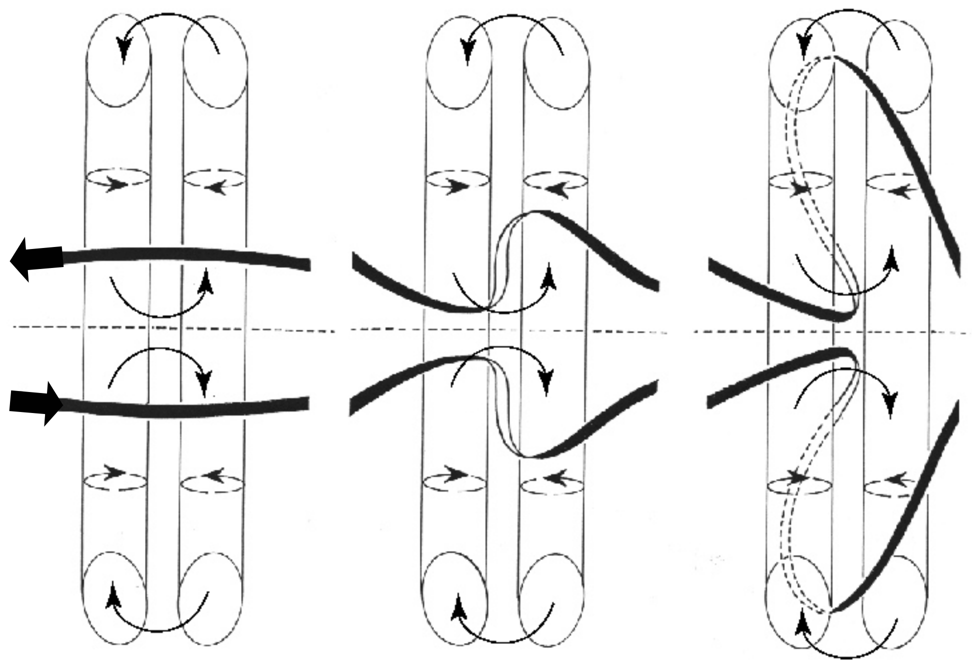


The columnar flow in rotating spherical shell convection is very helical, with negative helicity in the northern hemisphere and positive helicity in the southern hemisphere/

Toroidal - poloidal conversion in the benchmark dynamo



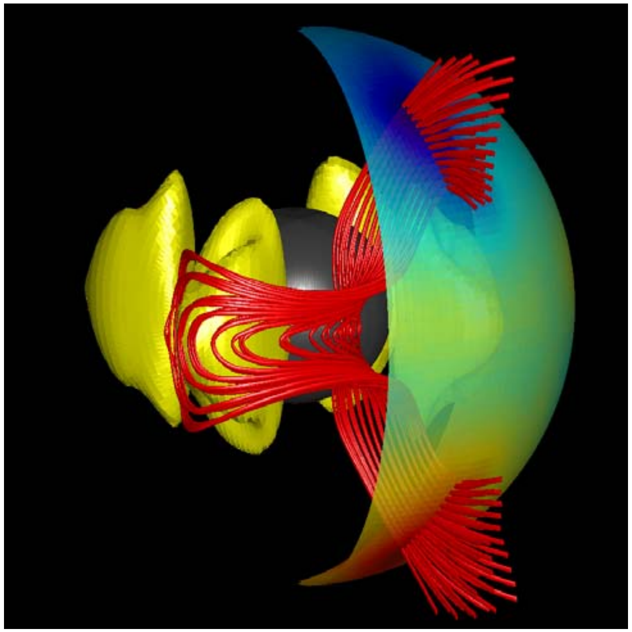
Axisymmetric field. Toroidal in color, poloidal as field lines



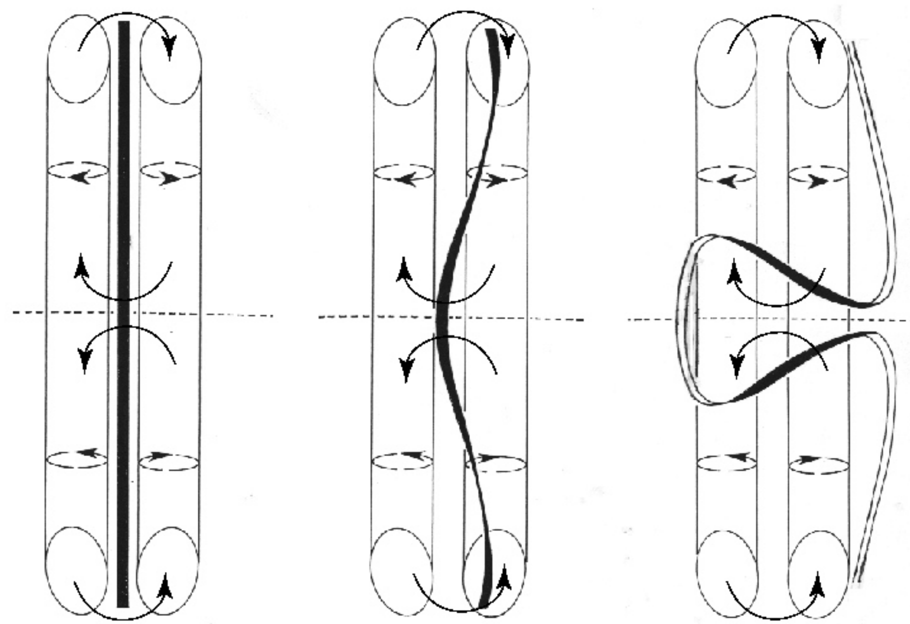
Cartoon showing advection and twisting of originally toroidal field lines (assuming frozen flux) by the helical columar flow, transforming them into a configuration with a strong poloidal dipole part

The dipole field is generated from the toroidal axisymmetric field by a macroscopic α-effect in the helical convection columns.

Poloidal - toroidal conversion in the benchmark dynamo



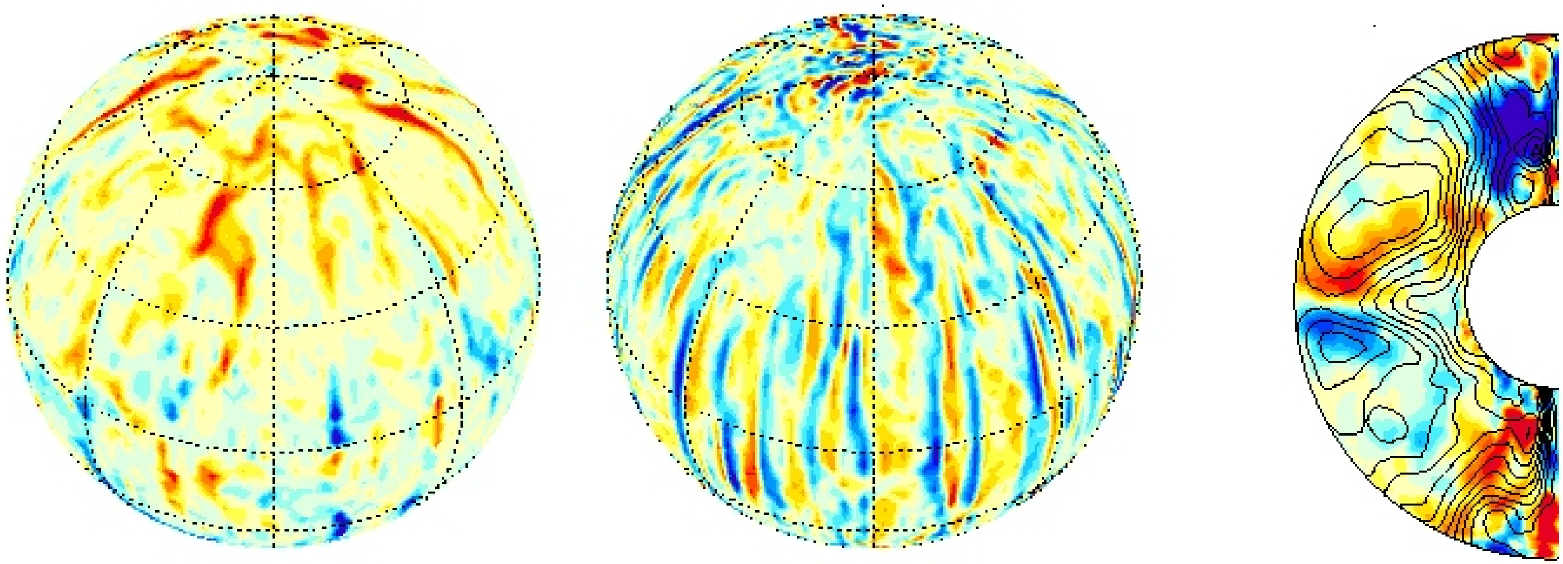
Bundle of field lines. Yellow: anticyclonic vortices.



Cartoon showing the evolution of a poloidal field line starting close to the tangent cylinder.

The axisymmetric toroidal field is generated by an α-effect in the helical columnar motion. In this dynamo, the ω-effect does not play a constructive role (α^2 instead of αω-type of dynamo). Note that in the cartoon the final configuration is the same (seen shifted by one column). Note also the similarity to the actual field lines in the numerical model.

A more Earth-like dynamo model

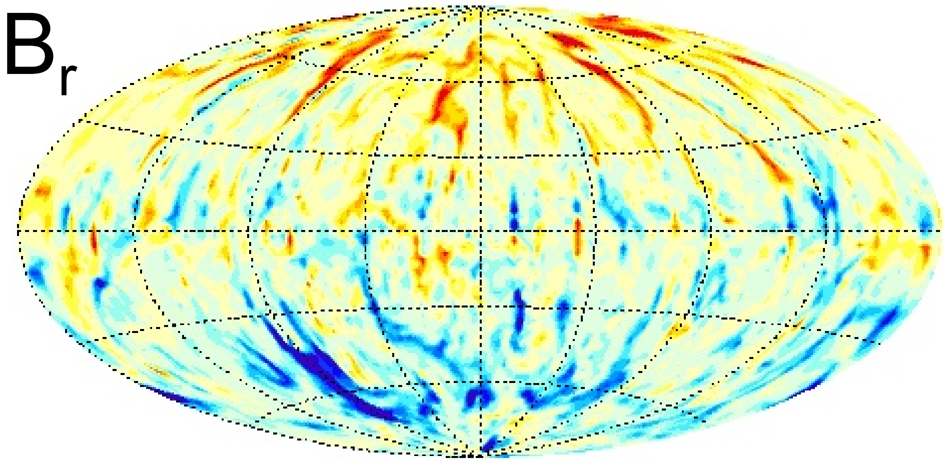


Radial magnetic field   Radial velocity   Axisymmetric field

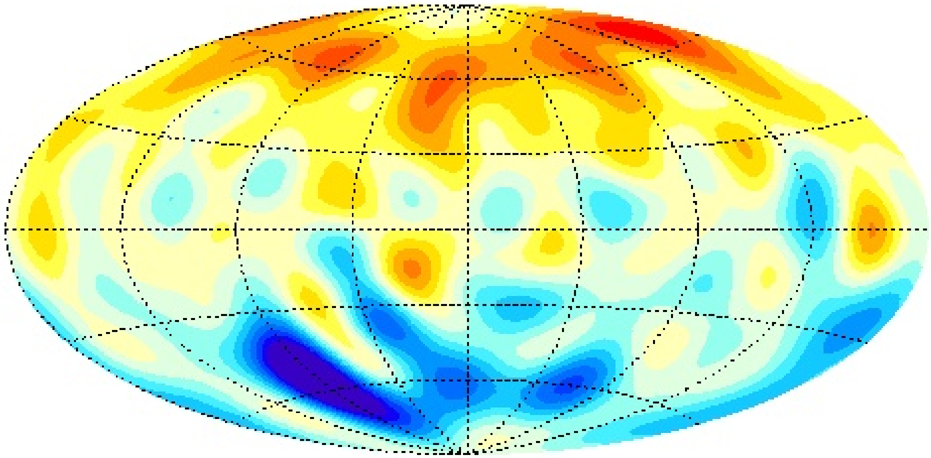
Ra = 1.2x10^{8} Pr = 1   E = 3x10^{-5} Pm = 2.5   Rm = 925

Dipole dominance in the exterior field despite small scale of magnetic flux concentrations.
North-south alignment of velocity structures $\Rightarrow$ (imperfect) convection columns
Strong upwelling flow at the pole.
Similar toroidal field structure as in simple model outside tangent cylinder $\Rightarrow$ same mechanism
Strong toroidal flux rings inside tangent cylinder generated by ω-effect

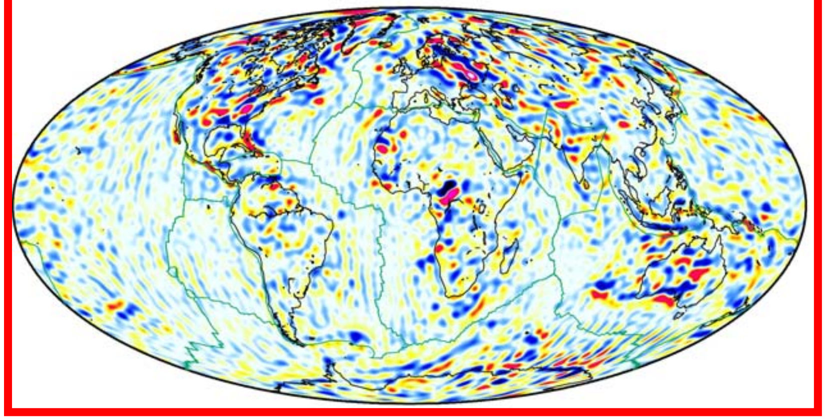
Field morphology compared with Earth's field



Dynamo model, full resolution

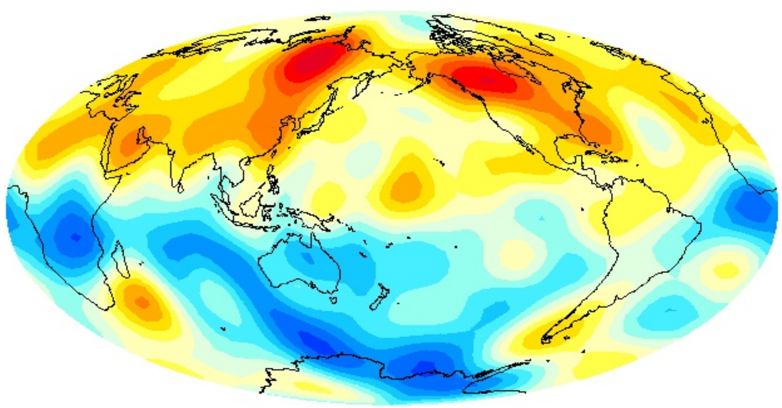


Dynamo model, filtered

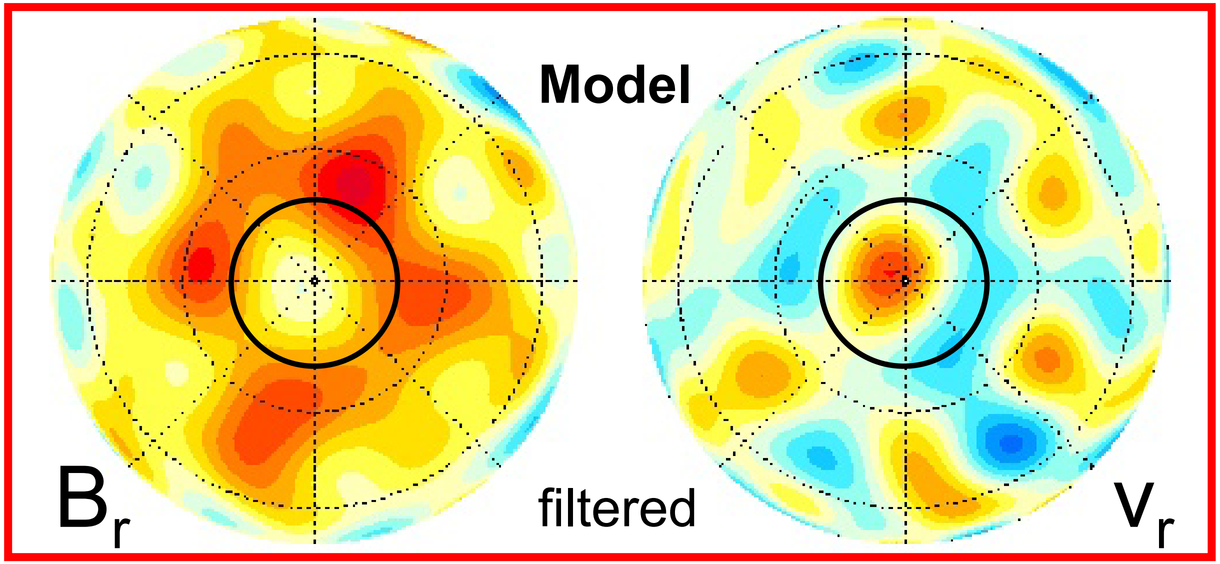


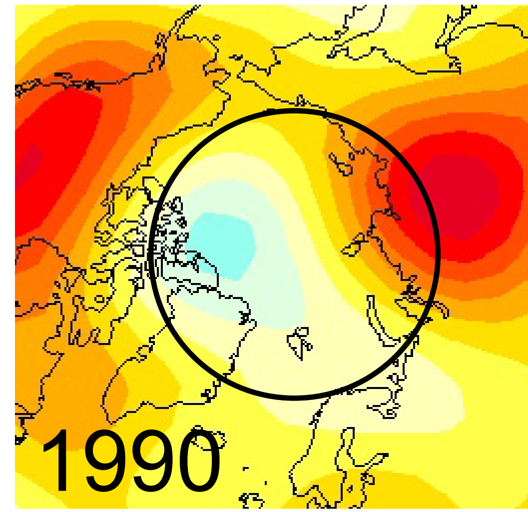
Crustal field masks small-scales of core field

Earth's field at core mantle boundary



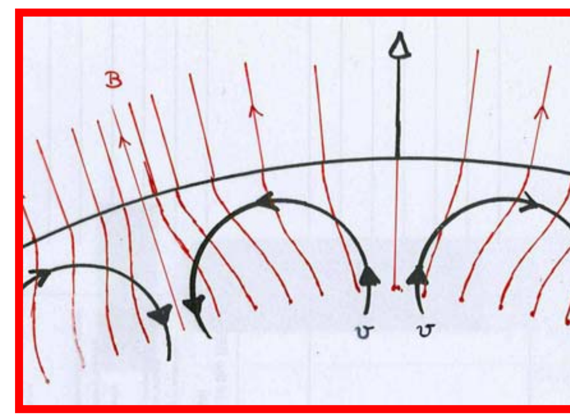
Field structure & core dynamics





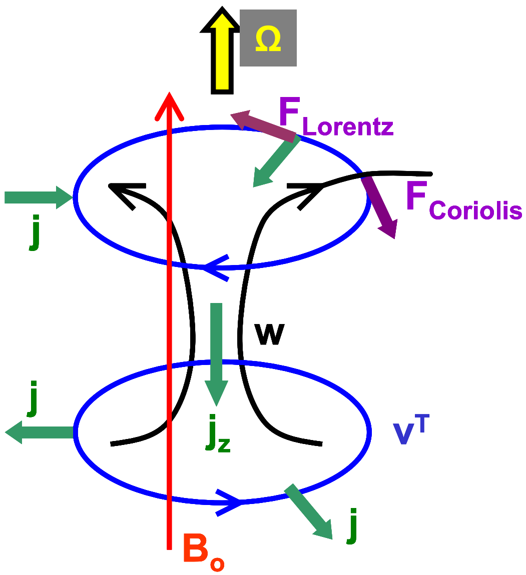
Earth‘s magnetic field at the CMB around the North pole

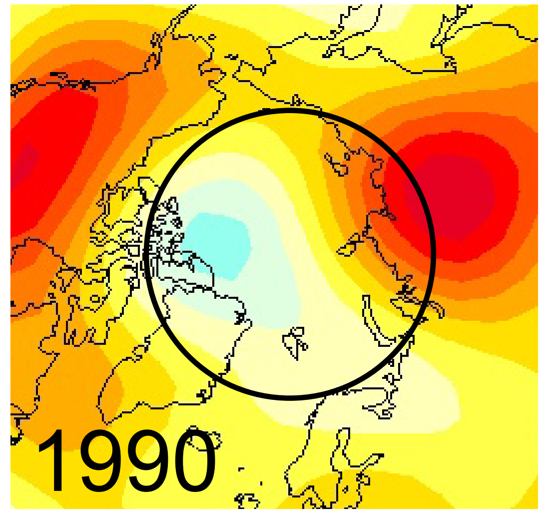
Polar view on the (filtered) model magnetic field at r_o and radial flow at $0.95r_o$. Circles indicate the tangent cylinder. In the model there is strong upwelling flow in the tangent cylinder and distributed downflow near and outside the tangent cylinder boundary. Magnetic flux concentrations correlate with downwelling and low flux with upwelling. This can be explained with the frozen flux assumption: Field lines are carried away by the divergent flow around upwellings and are collected by the convergent flow at downwellings. The field structures at Earth‘s North polar cap is consistent with such a flow pattern in the core.

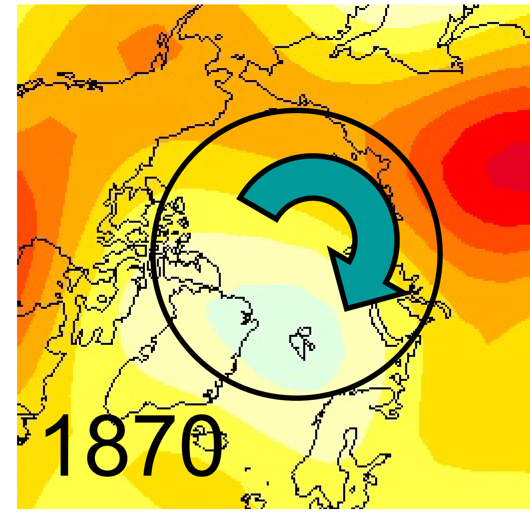


Advection of fieldlines

Field structure & core dynamics II







Flow in the tangent cylinder, where **Ω, g,** and the axial dipole field **B** are nearly parallel, may be an analogue in the core to the simple magnetoconvection model. Upwelling flow is associated with a clockwise vortex motion near the upper boundary (core-mantle boundary). The shift of the patch of low/inverse flux during 120 yr agrees with a clockwise rotation by 100°.

At the inner core boundary the vortex motion is anticlockwise (eastward). The solid inner core is coupled by electromagnetic forces to the flow near its boundary. Based on this effect a superrotation of 1-2°/yr of the inner core had been predicted. While initial seismic results seemed to support such rotation, more recent work arrives at lower rates.