8. EXPLOSIVE VOLCANISM

8.1

Pressure increases in the magma chamber due to tectonic movement, crystallization, gas release..., which eventually leads to disruption of the surrounding country rock and a potential eruption.

(e.g. H₂, Sparks and Turner, 1982 and 1983)

8.2

Density of wet magma as a function of the weight fraction of crystals for various total weight fractions of water $N$.
2. Dry magma ascent

i) Dry magmas ascend up a conduit very much like long gravity currents, under an interplay of fluid mechanics, thermodynamics and elasticity.

a) Fluid mechanics and thermodynamics

\[ T = T_C \quad @ t = 0 \]

HOT SHEAR FLOW IN CONTACT WITH A COLD WALL

- LOCAL RATE OF FREEZING \( \nu(z,t) \)
- INSTANTANEOUS DISTANCE FROM WALL \( y \)

\[ T_t - \nu T_y + \gamma T_z = \kappa T_{yy} \quad (y > 0) \quad \text{ADVECTION-DIFFUSION} \]

\[ T_t - \nu T_y = \kappa T_{yy} \quad (y < 0) \quad \text{DIFFUSION} \]

\[ L = -\kappa \frac{1}{c} \frac{d}{dy} \]

INTERFACIAL B.C.

\[ T = T_M \quad (y = 0) \]

\[ T = T_H \quad (y \to \infty \quad \text{OR} \quad t = 0, y > 0 \quad \text{OR} \quad z = 0, y > 0) \]

\[ T = T_C \quad (y \to -\infty \quad \text{OR} \quad t = 0, y < 0) \]

SOLVE SEMI-NUMERICALLY TO ALLOW FOR \( \gamma(t) \)

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(BRUCE AND H^2, 1989)

\[ L = F_{\text{rock}} - F_{\text{melt}} \]

\[ v > 0 \Rightarrow \text{FREEZING} \]

\[ v < 0 \Rightarrow \text{MELTING} \]

LARGE \( Z \), SMALL \( t \) \( \Rightarrow F_{\text{rock}} > F_{\text{melt}} \Rightarrow \text{FREEZING} \)

SMALL \( Z \), LARGE \( t \) \( \Rightarrow F_{\text{rock}} < F_{\text{melt}} \Rightarrow \text{MELTING} \)
b) Fluid mechanics and elasticity

2-D SOLUTION

\[ \frac{\dot{w}}{\dot{t}} + \frac{\Delta p}{3\eta} \frac{\dot{w}^2}{\dot{z}} = \frac{1}{3\eta} \frac{\dot{z}}{\dot{z}} \left( w^3 \frac{\dot{z}}{\dot{z}} \Delta p \right) \]

SOURCE FLUX \( q \) (constant)

\[ \text{BUOYANCY} \]

\[ w_\infty = \left( \frac{3\eta v}{3\eta g} \right)^{\frac{1}{2}} \]

ELASTICITY

\[ w_\infty \text{ with propagation rate } v = \frac{q}{2w_\infty} \]

solve numerically for \( w(s = z - vt) \)

\[ q = 2.5 \text{ m}^2 \text{s}^{-1} \Rightarrow w_\infty = 0.5 \text{ m}, \quad v = 2.5 \text{ ms}^{-1} \]

ii) For "wet" magmas, small exsolved vapour bubbles travel with the magma,

\[ v_b \sim 10^{-15} \text{cm s}^{-1} \ll v_m \sim 10^{10} \text{cm s}^{-1} \]

with \( v \) increasing by 10 for every 1 wt % of volatiles exsolved.

3. Wet magma ascent

Linear elasticity

\[ \beta : \text{ bulk modulus} \quad \frac{V dp}{\beta} = dV \quad (\ast) \]

Most general equation

\[ f(p ; z) dp = dV \quad (\ast \ast) \]
\[ Q_o = \left( \pi r_E^2 \right) \Delta p \approx \gamma \Delta p \approx 10^{-6} \Delta p \]

\[ p_o + \rho_w g_z \]

\[ p_b = p_o + \rho_w g H \approx 2 \times 10^8 \text{ Pa} \]

**Mass conservation:**
\[ \frac{d}{dt}(\rho V) = \rho \frac{dV}{dt} + V \frac{d\rho}{dt} = Q_I - Q_b = Q \] (1)

**Density relationship:**
\[ \rho = \rho\left[ \rho, T, x(T), N \right] \] (2)

\[ \frac{dV}{dt} + \frac{V}{\rho} \frac{d\rho}{dt} \frac{\partial \rho}{\partial T} = \frac{Q}{\rho} - \frac{V}{\rho} \frac{\partial \rho}{\partial T} \frac{dT}{dt} \] (3)

**Rock elasticity:**
\[ V dp = \beta_r dV \] (\( \beta_r \) rock bulk modulus \( \approx 10^{10} \text{ Pa} \)) (4)

\[ \left( \frac{V}{\beta_r} \right) \frac{d\rho}{dt} = \left( \frac{V}{\beta_r} \right) \frac{d\rho}{dt} \]

**Effective thermal expansion:**

**Solubility:**
\[ n = N - s \rho^{1/2} (1 - x) \] (5)

\( n \) mass fraction of exsolved volatiles; \( s \) solubility constant \( \approx 3 \times 10^{-6} \text{ Pa}^{1/2} \)

**Density relationship:**
\[ \rho = \left[ \frac{nPT}{p} + (1-n) \left( \frac{x}{\sigma_c} + \frac{1-x}{\sigma_m} \right) \right]^{-1} \] (6)
\[ C = \frac{1}{\beta_{\text{eff}}} \text{ (Pa}^{-1}) \]

\[ N = 5\% \]

\[ 3\% \]

\[ 1\% \]

\[ 1 \& 3\% \]

\[ x = 0.4 \]

\[ H (\text{km}) \]

\[ \beta_{\text{eff}} \text{ (Pa)} \]

\[ \rho \text{ (MPa)} \]

\[ \text{H2 \& Woods (2002, 2003)} \]
4. Steady conduit dynamics

Exit at sonic SPEED and OVERPRESSURE

High speed flow of ash and gas

Fragmentation Level

Magma becomes foam-like

Decompression of liquid magma

VENT

MAGMA CHAMBER

Equations for steady homogeneous flow in pipe

mass conservation \[ \rho u A = Q \] (1)

momentum conservation \[ \rho u \frac{du}{dz} = -\frac{dp}{dz} - \rho g - f \] (friction) (2)

density \[ \rho' = (1 - n)\rho_s + nRT/\rho \] (solid + gas) (3)

volatile content \[ n = n_0 - sp^{1/2} \] Henry's law (viscous liquid) (4)

constant \[ f \sim \frac{1}{2} \mu u/r^2 \] (turbulent gas) (5a)

\[ \sim 0.001 \rho u^2/r \] (5b)

void fraction \[ \phi = \left[ 1 + \frac{(1-n)p}{nRT\rho_s} \right]^{-1} \] (fracture formation at about \( \phi = 75\% \)) (6)
(1), (2), and (3) \[ \frac{dp}{dz} \left( 1 - \frac{u^2}{a^2} \right) = -\rho g - f \quad (7) \]

where sound speed \[ a^2 = \frac{dp}{dp} = a^2(\rho) \quad (8) \]

with \[ a = 0.95(n_0RT)^{1/2} \quad (9) \]

Integrate equations from

\[ p = \rho_0 \quad (z = 0) \quad \text{(in chamber)} \quad (10) \]

\[ p = \rho_e \quad \text{OR} \quad u = a \quad (\rho_e > \rho_0) \quad (z = H) \quad \text{(at surface)} \quad (11) \]

exit pressure
5. Crystals in conduits

mass conservation \( \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z} \rho w = 0 \) \( w(z, t) \): vertical velocity

momentum conservation \( \frac{\partial P}{\partial z} = -\rho g - \frac{8\mu w}{r_E^2} \)

crystal growth \( \frac{\partial x}{\partial t} + w \frac{\partial x}{\partial z} = 4\pi \Gamma \phi r^3 = (36\pi\phi)^{1/3} \Gamma x^{2/3} \)

number density of crystals \( \phi(z, t) \);

constant linear crystal growth rate \( \Gamma \)

boundary conditions \( z = 0 \) \( \frac{dP}{dt} = \frac{B}{V} (Q_I - Q_0) \); \( x = x_o \)
\( z = H \) \( p = p_{atm} \)
chamber pressure \( P \) vs. magma ascent velocity

8.17

Melnik (1999)

chamber pressure \( P \) vs. time

8.18

Melnik (1999)
6. Simple decompression phase over flat ground

Sonic speed \( a \approx 0.95(n_b RT)^{1/2} \) and overpressure \( 5 < \frac{p_e}{p_b} < 100 \)

\[
\nabla \cdot (\rho \mathbf{u}) = 0 \quad \text{(continuity)} \quad (1)
\]

\[
\rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p \quad \text{(momentum)} \quad (2)
\]

\[
\rho (\mathbf{u} \cdot \nabla) \left[ C_v T + p' \rho + u^2/2 \right] = 0 \quad \text{(enthalpy)} \quad (3)
\]

Integrating over a control volume \( V \)

\[
(1) \Rightarrow \quad \rho u A = Q \quad (4)
\]

\[
(2) \& (4) \Rightarrow \quad u_a = u_e + A \left( \frac{p_e}{p_b} - 1 \right) Q \quad (5)
\]

\[
\approx 1.8(n_b RT)^{1/2} \approx 250 - 400 \text{ ms}^{-1}
\]

\[
(3) \& (5) \Rightarrow \quad T_a \approx T_e \approx 10^3 \text{ K}
\]

7. The physics of eruption columns

Conversion of thermal energy to potential energy in dense, hot, decelerating jet.

If mixture becomes less dense than air before upward momentum exhausted

**BUOYANT PLUME**

If not, jet collapses

**GROUND HUGGING ASH FLOW**

Quantitative analysis in terms of entraining plume models of Morton, Taylor and Turner (1956)
Using standard atmospheric values

\[ H = 0.0082Q^{1/2} \]
\[ Q = \rho_e c_v e (T_e - T_a) \]

- **Q**: Thermal energy production rate (kW)

8. Important concepts

- Pressure increase in magma chambers
- Dry magmas ascend under influence of pressure release, fluid mechanics, thermodynamics and elasticity
- Wet magmas exsolve water vapour as they rise
- At fragmentation level, liquid film surrounding gas bubbles fracture and material evolves from a bubbly liquid (with solids) to an ash-laden gas (with small pockets of liquid) at around $\phi = 75\%$
- Eruption decompresses to enter atmosphere between 250 and 400 m s$^{-1}$
- Large base velocity and small flux produces buoyant plume, while small base velocity and large flux produces pyroclastic flows
- Energy in natural events greatly dominates that in those due to man (controlled or otherwise)
- 12 of the 16 largest volcanic eruptions in the last 200 years occurred at sites believed to be inactive
Lecture 8. Explosive Volcanism


