Parameter Dependence of Eastward-Westward Asymmetric Jets in Forced Barotropic 2D Turbulence on a β-plane

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1 Introduction

1.1 Background

- Zonal flow is dominated in forced 2D turbulence on a β-plane.
- alternating zonal jets in latitude.
  - When β is large, asymmetric zonal flow exists.
    Vallis & Maltrud(1993), Danilov & Gryanik(2004), Danilov & Gurarie(2004)
    - Eastward flow is narrower and faster than westward flow (see Figure 1(a))
    - saw-tooth pattern of the vorticity profile (see Figure 1(b)).

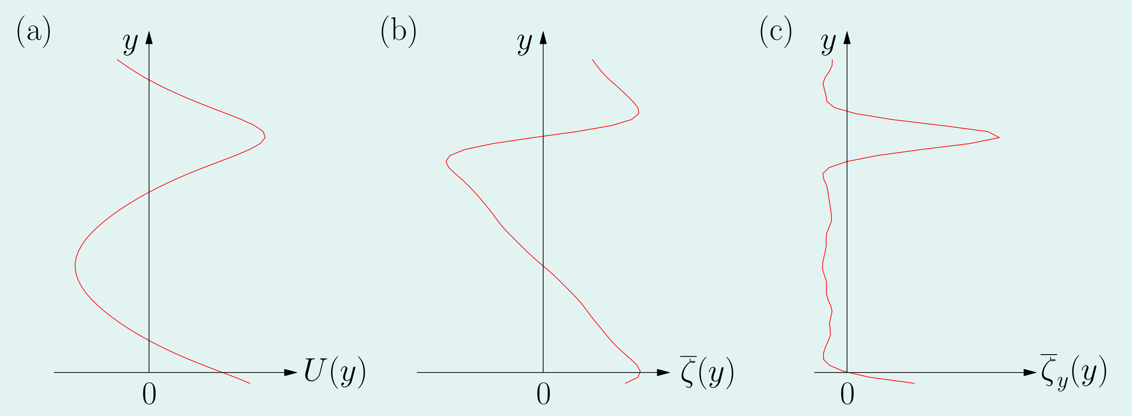

Figure 1. Three zonally averaged profiles calculated from a numerical experiment, (a) zonal velocity, (b)relative vorticity, (c) meridional gradient of relative vorticity.

- Mechanisms of producing the zonally asymmetric jet have not been well known yet. A possible mechanism have been proposed. (e.g., Smith & Lee, 2006)
- Parameter dependence of zonal asymmetry of zonal jets have not been known yet. → the present study (In the present study, we concentrate our attention on the saw-tooth profile of zonally averaged vorticity.)

2 Definition of zonal asymmetricity

We define zonal asymmetricity as the ratio of zonally averaged relative vorticity gradient: $r := \bar{\zeta}_{y+}/|\bar{\zeta}_{y-}|$.

$$\bar{\zeta}_{y+} := \frac{1}{L_+}\int_{\bar{\zeta}_y>0} \bar{\zeta}_y\, dy, \qquad L_+ := \int_{\bar{\zeta}_y>0} dy$$

$$\bar{\zeta}_{y-} := \frac{1}{L_-}\int_{\bar{\zeta}_y<0} \bar{\zeta}_y\, dy, \qquad L_- := \int_{\bar{\zeta}_y<0} dy$$

According to Danilov & Gurarie(2004), we approximate the vorticity profile(Figure 1(b)) to an idealized saw-tooth pattern as Figure 2(b). Then, the vorticity gradient has piecewise constant profile(see Figure 2(c)). In this approximation, $\bar{\zeta}_{y+}$ and $\bar{\zeta}_{y-}$ defined above correspond to those in Figure 2(c). Moreover, if the jet wavenumber k_j is known, $l_+ = L_+/k_j$ and $l_- = L_-/k_j$ correspond to the width of regions $\bar{\zeta}_y > 0$ and $\bar{\zeta}_y < 0$, respectively (see Figure 2(c)).

Since $\int \bar{\zeta}(y)dy = 0$, $L_+\bar{\zeta}_{y+} = -L_-\bar{\zeta}_{y-}$. This leads to $r = l_-/l_+$.

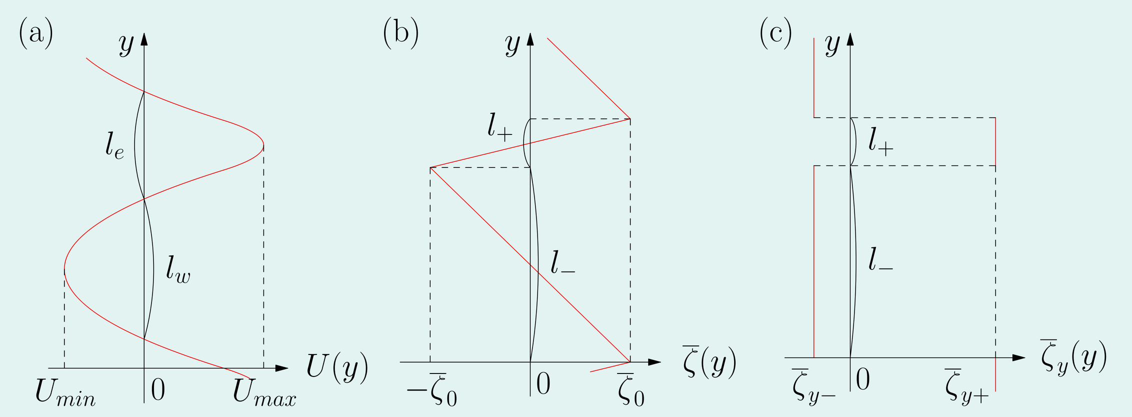

Figure 2. Three zonally averaged profiles calculated as vorticity gradient is piecewise constant, (a) zonal velocity, (b) relative vorticity, (c) meridional gradient of relative vorticity.

Parameter dependence of r is determined by l_+ and l_-.
→ We explore a parameter dependence of l_+, l_- and r.

3 Basic equation and conditions of numerical experiment

3.1 Basic equation

barotropic vorticity equation on a β-plane

$$\frac{\partial \zeta}{\partial t} + \frac{\partial \psi}{\partial x}\frac{\partial \zeta}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial \zeta}{\partial x} + \beta\frac{\partial \psi}{\partial x} = D\zeta + F \qquad (1)$$

- $\zeta = \nabla^2\psi$: relative vorticity
- ψ: stream function
- β: the meridional gradient of Coriolis parameter
- D: dissipation operator
- F: forcing

3.2 Conditions of Numerical Experiments

- $D = -\lambda_n(-\nabla^2)^{-n} - \nu_m(-\nabla^2)^m$
- F: Markovian forcing, homogeneously given in the range where total wavenumber k is $98 \le k \le 102$.
- nearly constant energy input rate ϵ
- initialized at zero vorticity
- doubly periodic boundary condition, and domain size is $2\pi \times 2\pi$
- spectral method
- 2nd order Adams-Bashforth time stepping scheme
- grid point number: 512^2
- truncation wavenumber: 170
- $\lambda_2 = 50, 123, 300$, $\lambda_1 = 5$
- β varies from 0 to 600.

4 Results

4.1 Results

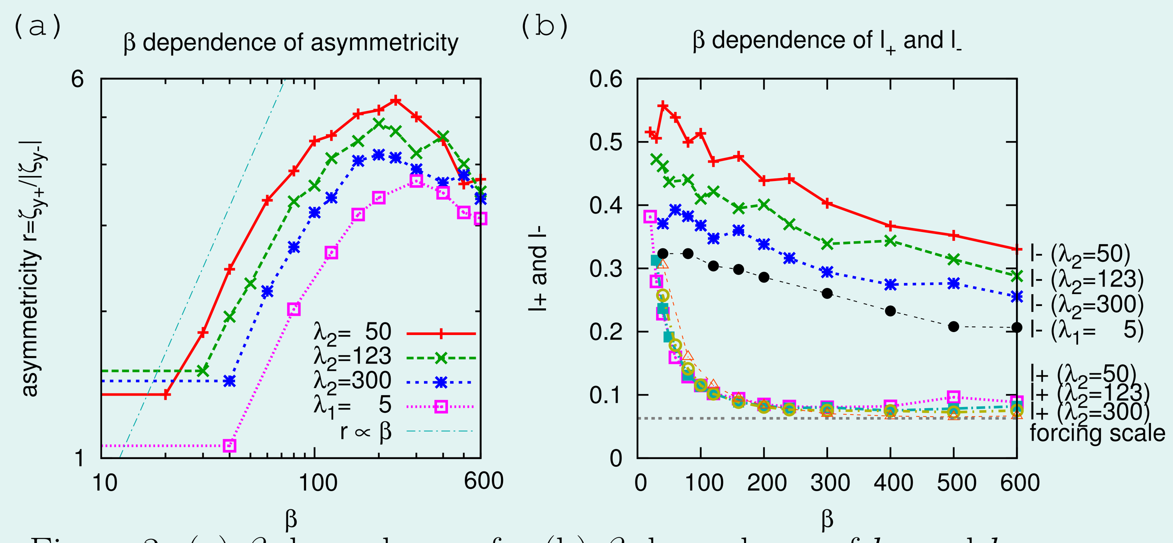

Figure 3: (a) β dependence of r (b) β dependence of l_+ and l_-.

- features of β dependence of r (Figure 3(a))
  - r increases as β increases in $0 \le \beta \le 200$.
  - r decreases as β increases in $\beta \ge 200$.
- features of β dependence of l_+ (Figure 3(b))
  - l_+ greatly decreases in $0 \le \beta \le 200$.
  - l_+ is nearly equal to the forcing scale l_f in $\beta \ge 200$.
- features of β dependence of l_- (Figure 3(b))
  - l_- monotonically decreases in all ranges of β.
  - l_- less decrease than l_+.

4.2 Why r has an upper bound?

The sign of the variation of r depending on β changes at $\beta \approx 200$.

- r increases in $\beta \le 200$,
  - l_- slightly decreases.
  - l_+ greatly decreases.
- r decreases in $\beta \ge 200$
  - l_- slightly, monotonically decreases.
  - l_+ is nearly equal to l_f.

Therefore r has an upper bound at $\beta_c \approx 200$.

5 Comparison of l_+ and l_- with pre-existing length scales

5.1 Background

Three characteristic scales in two-dimensional turbulence on a β-plane.

- Rhines scale, $k_{\mathrm{Rh}} = \sqrt{\beta/U_{\mathrm{rms}}}$
- Holloway-Hendershott scale, $k_{\mathrm{HH}} = \beta/\zeta_{\mathrm{rms}}$
- Vallis-Maltrud scale, $k_{\mathrm{VM}} = \left(\frac{\beta^3}{(0.75C_k)^{3/2}\epsilon}\right)^{1/5}$

It is well-known that Rhines scale can predict $l = l_+ + l_-$ very well.

Question: Can Rhines and HH and VM scales predict l_+ and l_-?

5.2 locally defined characteristic wavenumber

We define local characteristic wavenumbers in the region where $\bar{\zeta}_y > 0$, and the region where $\bar{\zeta}_y < 0$.

We write effective β in each region for the simplicity,

$$\beta_+ := \beta + \bar{\zeta}_{y+}, \text{ effective } \beta \text{ at } \bar{\zeta}_y > 0,$$
$$\beta_- := \beta + \bar{\zeta}_{y-}, \text{ effective } \beta \text{ at } \bar{\zeta}_y < 0.$$

5.2.1 Rhines scale

We define local Rhines scale, using effective β and difference zonal velocity from locally defined mean zonal velocity,

$$k_{\mathrm{Rh}+} := \sqrt{\frac{\beta_+}{U'_{+\mathrm{rms}}}}, \qquad k_{\mathrm{Rh}-} := \sqrt{\frac{\beta_-}{U'_{-\mathrm{rms}}}}. \qquad (2)$$

where

$$U'_+ := U(y) - \frac{1}{L_+}\int_{\bar{\zeta}_y>0} U(y)dy, \qquad (3)$$

$$U'_- := U(y) - \frac{1}{L_-}\int_{\bar{\zeta}_y<0} U(y)dy. \qquad (4)$$

5.2.2 Holloway-Hendershott scale

We define local Holloway-Hendershott scale,

$$k_{\mathrm{HH}+} := \frac{\beta_+}{\zeta_{+\mathrm{rms}}}, \qquad k_{\mathrm{HH}-} := \frac{\beta_-}{\zeta_{-\mathrm{rms}}}, \qquad (5)$$

where $\zeta_{\mathrm{rms}} = \zeta_{+\mathrm{rms}} = \zeta_{-\mathrm{rms}}$.

5.2.3 Vallis-Maltrud scale

We define local Vallis and Maltrud scale in each region,

$$k_{\mathrm{VM}+} := \left(\frac{\beta_+^3}{(0.75C_k)^{3/2}\epsilon}\right)^{1/5}, \qquad k_{\mathrm{VM}-} := \left(\frac{\beta_-^3}{(0.75C_k)^{3/2}\epsilon}\right)^{1/5}. \qquad (6)$$

We assume that energy input rate ϵ is same in two regions.

5.3 Results

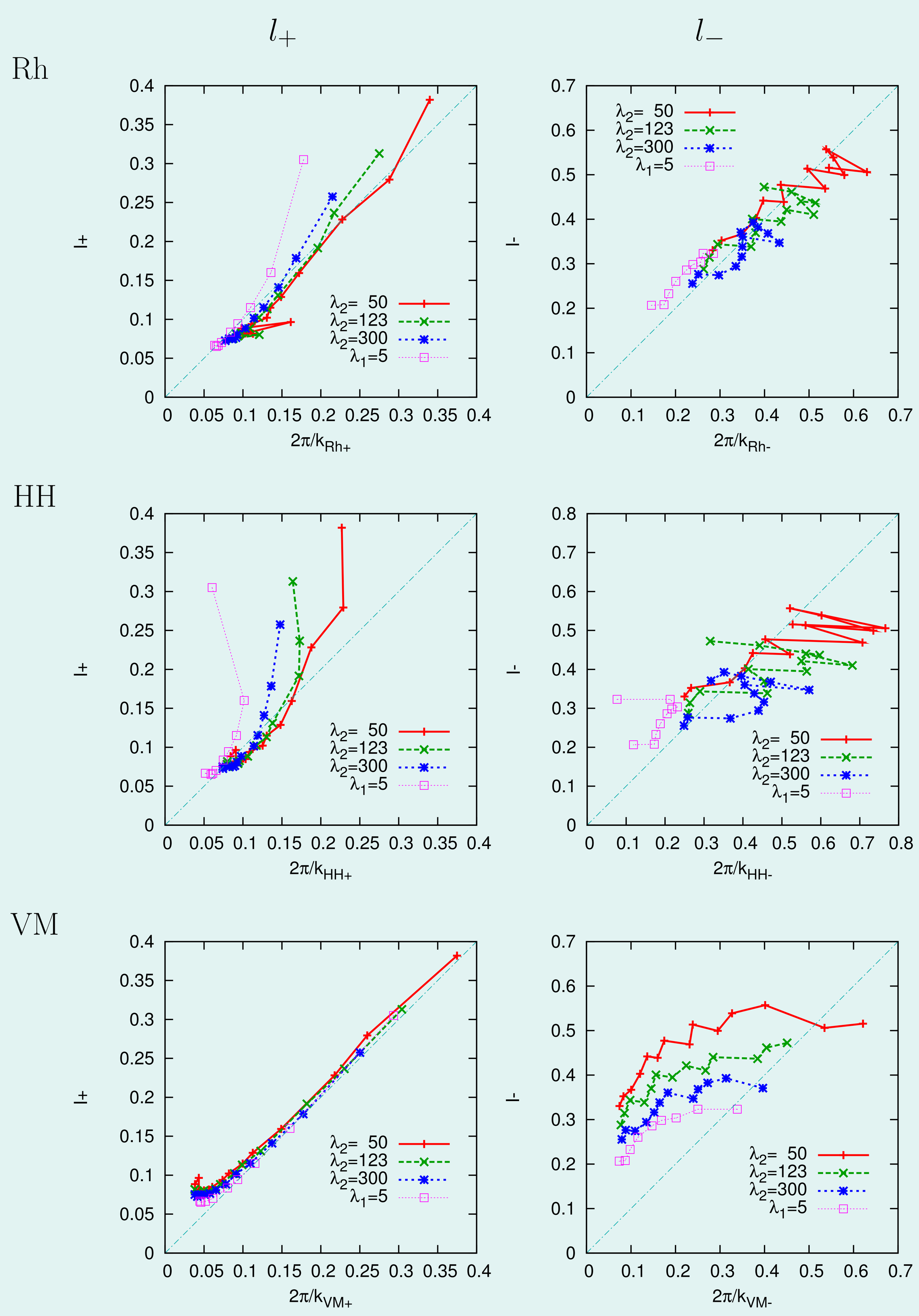

Figure 4: Comparison of local characteristic scales and l_+ and l_-.

- local Vallis and Maltrud scale can predict l_+ very well.
  - l_+ can be interpreted as a scale dividing the wave and turbulence regimes.
- local Rhines scale can give relatively good estimate to l_+ and l_-.

6 Summary

We examine the saw-tooth profile of the zonally averaged vorticity in forced 2D turbulence on a *beta*-plane, by analyzing the results on numerical simulation.

- We define eastward-westward asymmetricity r as $r := l_-/l_+$.
- We investigate parameter dependence of l_+, l_- and r.
  - When β is sufficiently large, $l_+ \approx l_f$.
  - l_- is less decrease than l_+. → This leads to increase of r.
- We investigate whether locally defined characteristic scales can predict l_+ and l_-.
  - local Vallis and Maltrud scale can predict l_+ very well.
  - local Rhines scale can give relatively good estimate to l_+ and l_-.

References

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