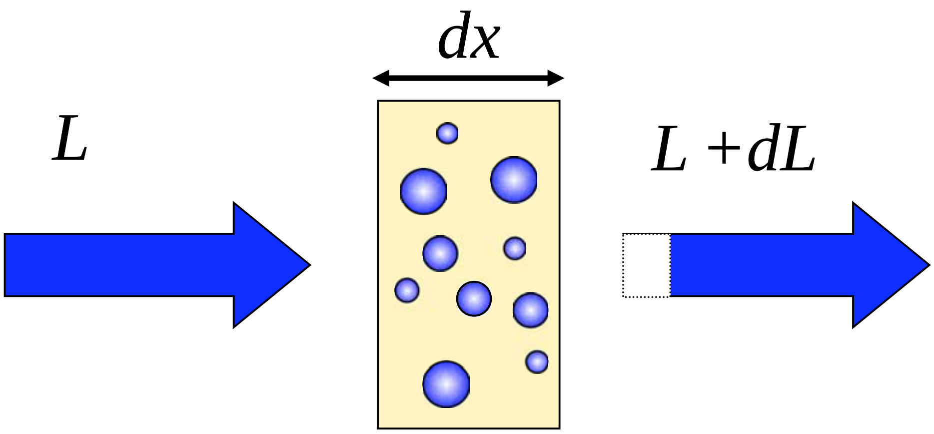
2. Radiation interaction with materials

Lambert-Beer's law: Differential form

- Monochromatic radiation（単色の光）
- (Volume) Extinction coefficient (消散係数)　[e] = 1/m

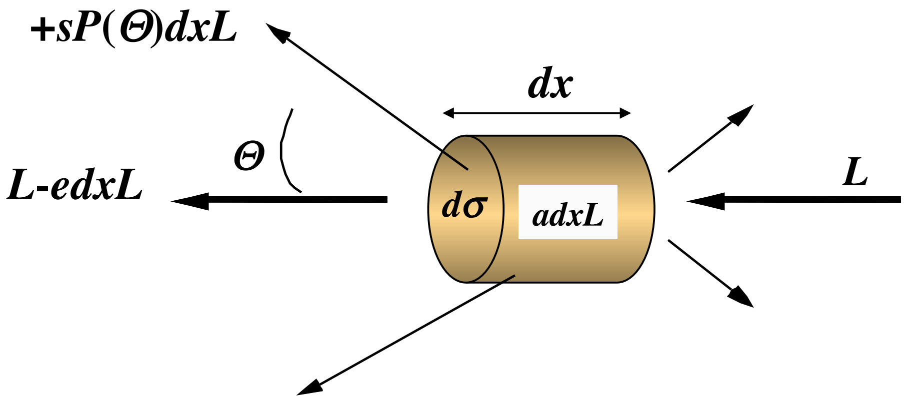
$$dL = -eLdx$$



Scattering phenomenon（散乱現象）

- Scattering of radiation to other angles
- Scattering angle（散乱角）: Θ
- Scattering coefficient（散乱係数）: s (m^{-1})
- Scattering phase function（散乱位相関数）: $P(\Theta)$
- Direct radiance: $dL = -edxLd\Omega$
- Scattered radiance: $dL = sP(\Theta)dxLd\Omega$

$$\int_{4\pi} P(\Theta)d\Omega = 2\pi\int_{-1}^{1} P(\Theta)d\cos\Theta = 1$$



Absorption and scattering

- Absorption and scattering phenomena with coefficients for absorption and scattering: a (1/m) and s (1/m)
- Energy conservation: $e\,Ldx = a\,Ldx + s\,L\,dx$ leads $e = a + s$
- Single scattering albedo: $\omega = s/e$
- Particle system composed of (equivalent) spheres with radius r with a number density N ($1/m^3$)
- Geometrical cross section: C_{geo}
- Optical cross sections (coefficients) for absorption, scattering and extinction: C_{abs}, C_{sca}, C_{ext}
- Efficiency factors: only depend on the size parameter $\alpha = kr$

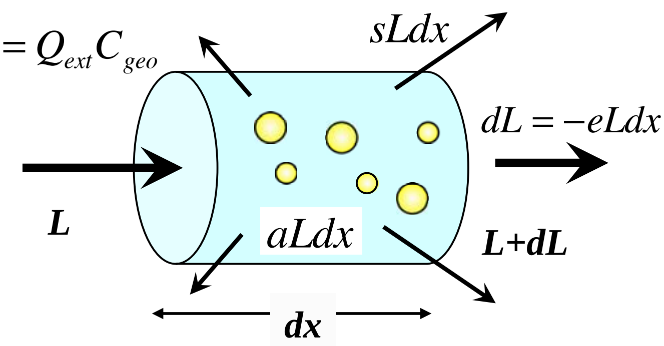
$$C_{geo} = \pi r^2$$

$$C_{abs} = Q_{abs}C_{geo}, \quad C_{sca} = Q_{sca}C_{geo}, \quad C_{ext} = Q_{ext}C_{geo}$$

$$C_{ext} = C_{abs} + C_{sca}$$

$$\alpha = kr = \frac{2\pi r}{\lambda}$$

$$a = C_{abs}N, s = C_{sca}N, e = C_{ext}N$$



Lambert-Beer's law: Integral form

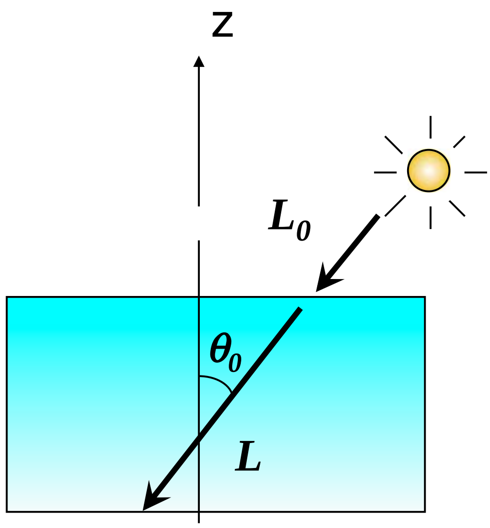
- Direct radiation (直達放射)
- Optical thickness (光学的厚さ), depth [*m*]x[1/ *m*]=[1]
- Optical airmass m
- Solar zenith angle (太陽天頂角)

$$dL = -eLdz / \cos\theta_0$$

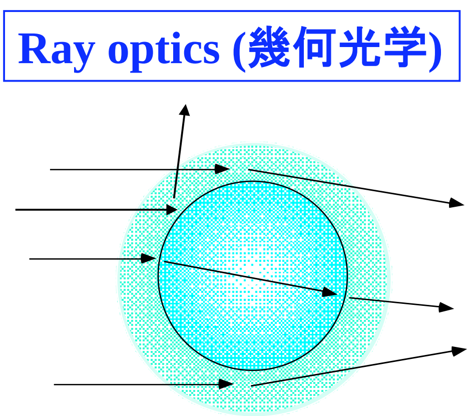
$$L = L_0 \exp(-\tau m)$$

$$T \equiv L / L_0 = \exp(-\tau m)$$

$$\tau = \int_z^\infty e(z)dz, \quad m = 1/\cos\theta_0$$



Large particle limit



- Large particle limit
  - Refraction and diffraction

$$Q_{ext} \to 2,$$

$$Q_{sca} \to 2\,(\omega = 1), \quad \to 1\,(\omega < 1)$$

$$e = Q_{ext} C_{geo} N = 2\pi r^2 N$$

$$\tau = e\Delta x$$

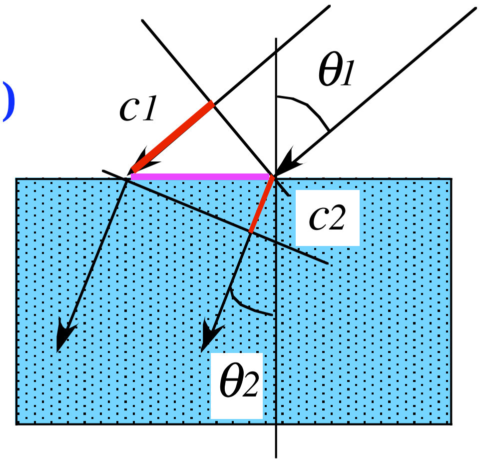
$$T = \exp(-m\tau)$$

r (micron)	10
N (particles/cc)	100
e (/m)	0.0628
dx (m)	100
tau	6.28
Solar zenith angle (de	60
Optical airmass	2.00
Direct transmittance	3.55E-06

- But, why is the cloudy sky so bright?

Refractive index

- Complex refractive index (複素屈折率)
- Snell's law
- Lambert absorption coefficient



$$E = E_0\, e^{i(\omega t - \tilde{m} k_0 x)}$$

$$\tilde{m} = m_r - i m_i$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2} = \tilde{m}_{12}, \quad \omega = k_1 c_1 = k_2 c_2, \quad k_2 = \tilde{m}_{12} k_1$$

$$|E|^2 = \left|E_0 e^{i(\omega t - \tilde{m} k_0 x)}\right|^2 = |E_0|^2\, e^{-\Lambda x}$$

$$\Lambda = 2 m_i k_0 = \frac{4\pi m_i}{\lambda}$$

wavelength	0.5 micron
mi	-0.01
Lambert abs	-251.2 for 1mm thickness

Small particle limit

Rayleigh (1871)
van de Hulst (1957)

- Small dielectrics; Rayleigh scattering (Dipole scattering): $r^6\lambda^{-4}$
- Loosing the efficiency of scattering
- Infrared region: Absorption is dominant

$$\alpha = ka = \frac{2\pi a}{\lambda}$$

$$Q_{ext} = \varepsilon_1\alpha + \varepsilon_3\alpha^3 + \varepsilon_4\alpha^4$$

$$Q_{sca} = s_4\alpha^4, \quad \text{if } \alpha < 1.5$$

$$s \approx s_4\alpha^4\pi r^2 N = s_4\pi(2\pi)^4 r^6\lambda^{-4} N$$

$$m_i = 0: \quad \varepsilon_4 = s_4, \omega = 1$$

$$m_i > 0: \quad \omega \to 0, \quad e \propto V$$

$$\alpha = (m_r^2 + m_i^2)^2, \quad \beta = m_r^2 - m_i^2, \quad \gamma = m_r m_i$$

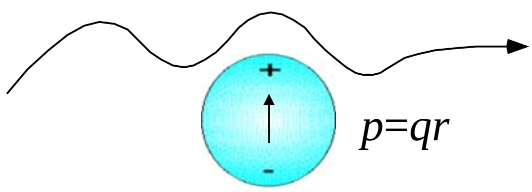
$$z_1 = \alpha + 4\beta + 4, \quad z_2 = 4\alpha + 12\beta + 9$$

$$\varepsilon_1 = \frac{24\gamma}{z_1}, \quad \varepsilon_3 = \gamma\left[\frac{4}{15} + \frac{20}{3z_2} + \frac{24}{5z_1^2}(7\alpha + 4\beta - 20)\right]$$

$$\varepsilon_4 = \frac{8}{3z_1^2}\left[(\alpha + \beta - 2)^2 - 36\gamma^2\right]$$

$$s_4 = \frac{8}{3z_1^2}\left[(\alpha + \beta - 2)^2 + 36\gamma^2\right]$$

もうひとつの導出方法
Dipole scattering
(双極子散乱)



Small particle limit (2)

$$m_r = 1 + \Delta m, \quad m_i = 0$$

$$\alpha = m_r^4 = 1 + 4\Delta m, \quad \beta = m_r^2 = 1 + 2\Delta m, \quad \gamma = 0$$

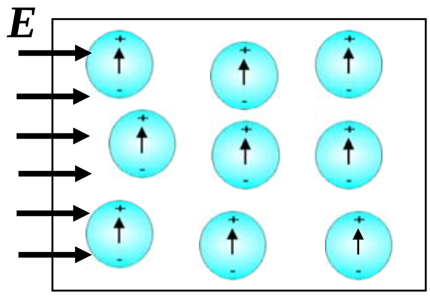
$$z_1 = 1 + 4\delta + 4 + 8\delta + 4 = 9 + 12\delta \approx 9$$

$$s_4 = \frac{8}{3z_1^2}(\alpha + \beta - 2)^2 = \frac{2^5}{3^3}\Delta m^2$$

$$C_{sca} \approx s_4 x^4 \pi r^2 = \frac{2^5}{3^3}\Delta m^2 \pi (2\pi)^4 \lambda^{-4} r^6$$

$$N = 1 / (\frac{4\pi}{3} r^3)$$

$$C_{sca} = \frac{2^5}{3^3}\Delta m^2 \pi (2\pi)^4 \lambda^{-4} (\frac{3}{4\pi N})^2 = \frac{32\pi^3}{3\lambda^4}(\frac{\Delta m}{N})^2$$



Rayleigh scattering

- **Refractive index of air** $m - 1 = 2.9\text{E-4}(1atm, 0C)$

$$C_{sca} \approx \frac{32\pi^3}{3\lambda^4}(\frac{m-1}{N})^2 = \frac{3.84\text{E-32}}{\lambda_{micron}^4} \quad (m^2)$$

$$N = N_a n / V = N_a P / RT = 6.02E23 * 1.013E5 / 8.31 / 273 = 2.69E25(/m^3)$$

- **Column air molecule number**

$$N_{air} = 1.013E5(N/m^2) / 9.8(m/\sec^2) * 6.02E23 / 0.029 = 2.1E29(/m^2)$$

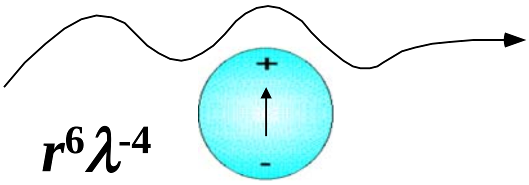
- **Rayleigh optical thickness of the atmosphere**

$$\tau_R = N_{air} C_{sca} = \frac{32\pi^3 N_{air}}{3\lambda^4}(\frac{m-1}{N_{1atm}})^2 = \frac{0.0081}{\lambda(\mu m)^4}$$

$$\tau_R = 0.00864 \lambda^{-(3.916 + 0.074\lambda + 0.050/\lambda)} P_{atm}$$

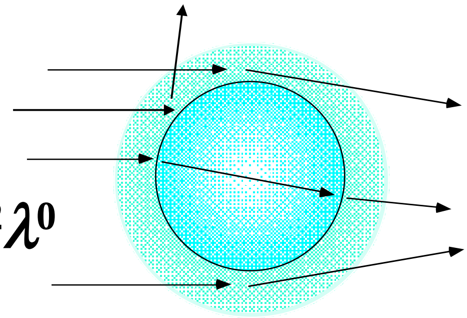
Multipole moments by dielectrics

Small particle limit: Rayleigh scattering



$r^6\lambda^{-4}$

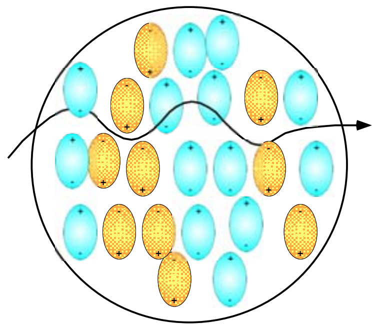
Large particle limit: Geometrical optics



$r^2\lambda^0$

Large dielectric sphere: Mie scattering theory (1908)

100 year anniversary in 2008!

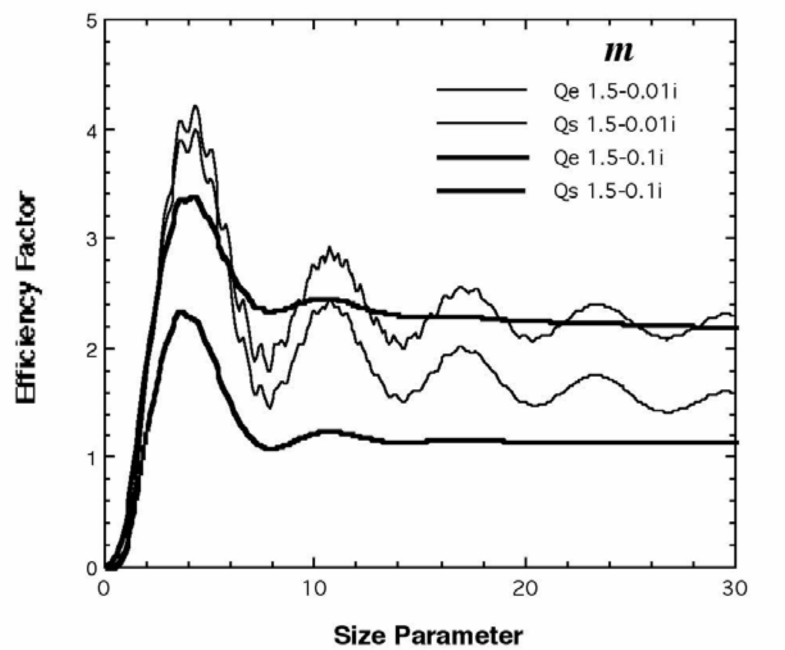


Efficiency factors

- **Efficiency factors**
- **Critical size parameter**

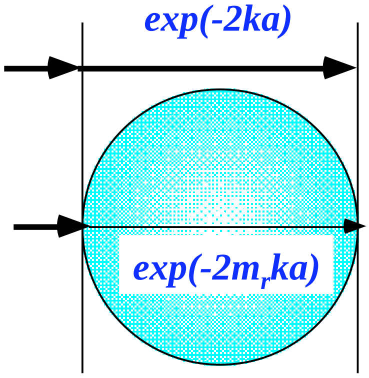
$$\alpha = kr = \frac{2\pi r}{\lambda}$$

$$C_{ext} = Q_{ext}\pi r^2, \quad C_{abs} = Q_{abs}\pi r^2, \quad C_{sca} = Q_{sca}\pi r^2$$

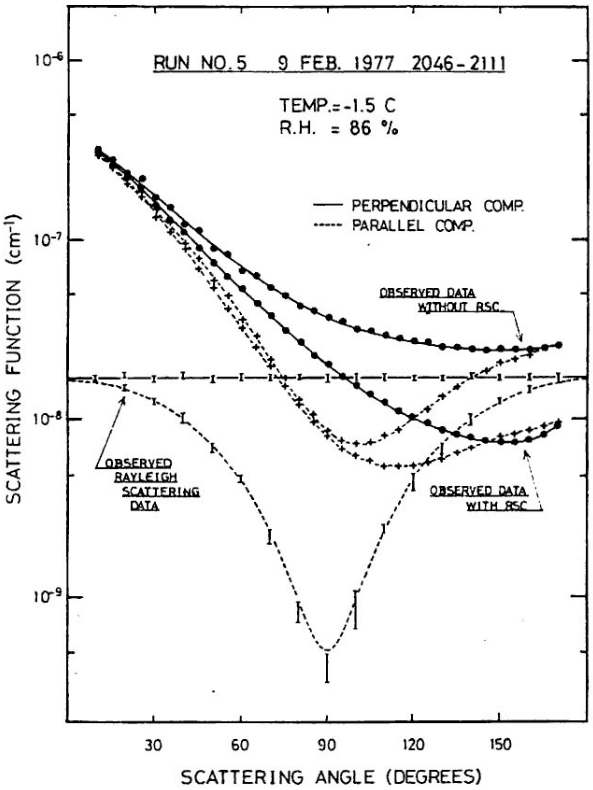


$$\alpha_c = \frac{\pi}{2(m_r - 1)}$$

$$2m_r ak - 2ak = \pi$$



Contribution of atmospheric molecules



- **Rayleigh scattering**
- **Gaseous absorption (water vapor, Ozone)**

$$\tau_R = 0.00864\lambda^{-(3.916+0.074\lambda+0.050/\lambda)} P_{atm}$$

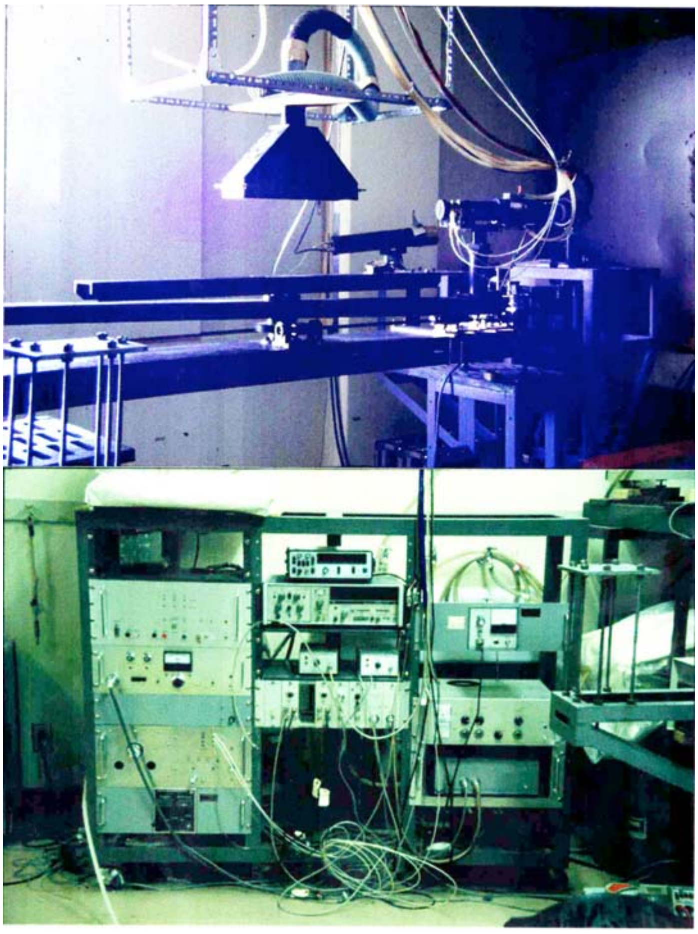
$$\tau = \tau_{aerosol} + \tau_{molecule}$$

$$\omega\tau = \omega_a\tau_a + \omega_m\tau_m$$

$$\omega\tau P(\Theta) = \omega_a\tau_a P_a(\Theta) + \omega_m\tau_m P_m(\Theta)$$

$$P_m(\Theta) \approx \frac{3}{16\pi}(1+\cos^2\Theta)$$

Tanaka et al. (1983)



Polar nephelomter

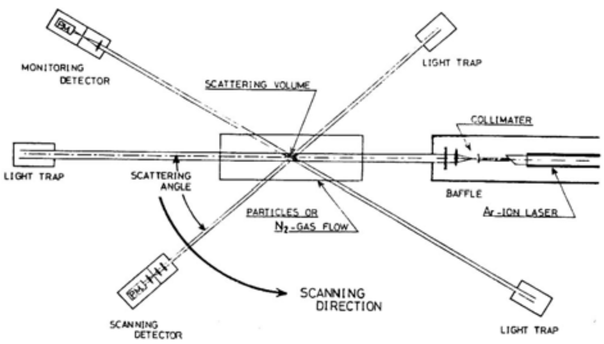
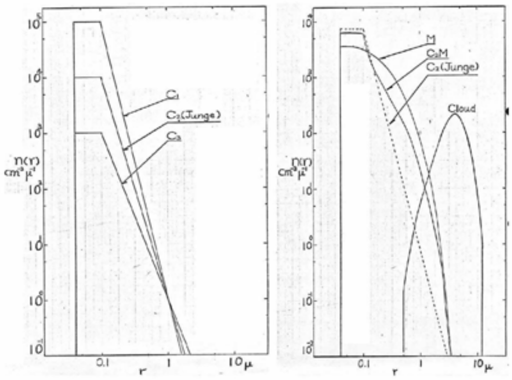


Fig. 1 Schematic diagram of polar-nephelometer.

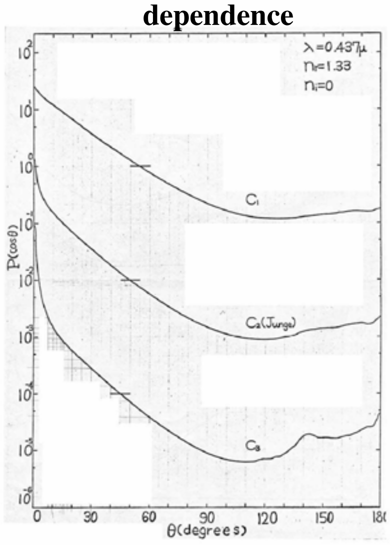
Takamura and Tanaka (1978)

Phase function characteristics

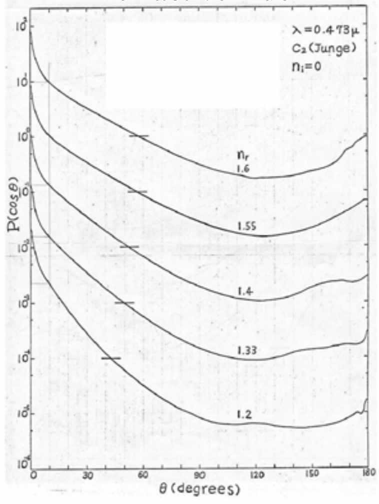
K. Sato (1973)



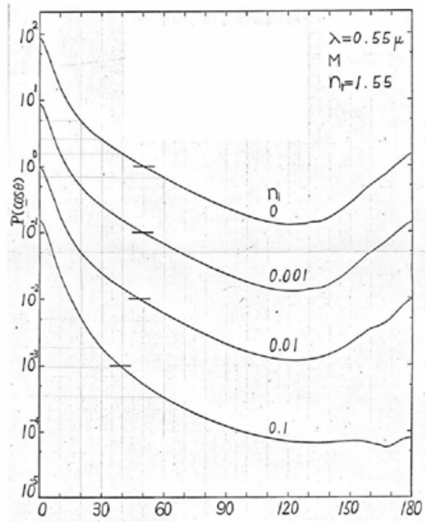
Size distribution dependence



Real part of refractive index



Imaginary part



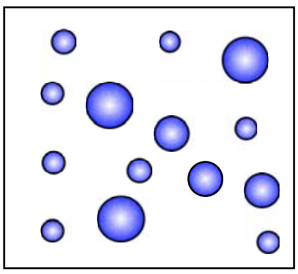
Particle size distribution（粒径分布）

- **Size distribution, Size spectrum**

$$v(\ln r)d\ln r = \frac{4\pi r^3}{3}n(r)dr, \quad v(\ln r) = \frac{4\pi r^4}{3}n(r)$$

- **Power law size distribution**

$$n(r) = \begin{cases} Cr^{-p} & if \;\; r > r_0 \\ Cr_0^{-p} & if \;\; r \le r_0 \end{cases}$$



  - **Junge distribution: *p*= 4**

- **Log-normal distribution**

$$n(r) = \sum_n \frac{N_n}{\sqrt{2\pi}\sigma_n}\exp\{-\frac{1}{2}(\frac{\ln r - \ln r_{m,n}}{\sigma_n})^2\}$$

- **Modified Gamma distribution**

$$n(r) = C\,r^{\alpha}e^{-\beta r^{\gamma}}$$

Observed size distributions

Pasceri, R.E., and S.K. Friedlander (1965)

$$N(r) = \int_r^\infty n(r)\,dr$$

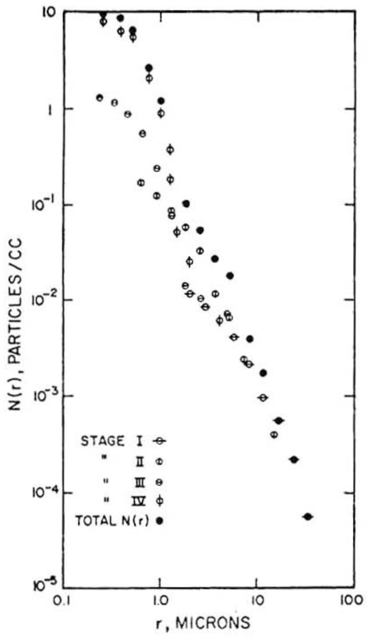


FIG. 1. Run 1 made outdoors in Baltimore 14 February 1961. Cumulative data for individual impactor stages are combined to give cumulative distribution for the run.

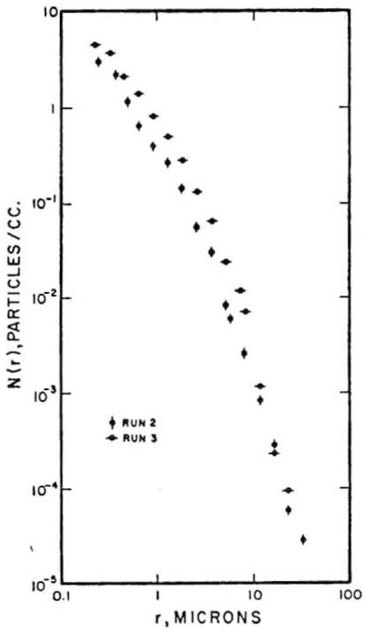


FIG. 2. Runs 2 and 3 made outdoors in a residential area outside Baltimore 30 June 1961 and 1 July 1961. The results are close to those shown in Fig. 1.

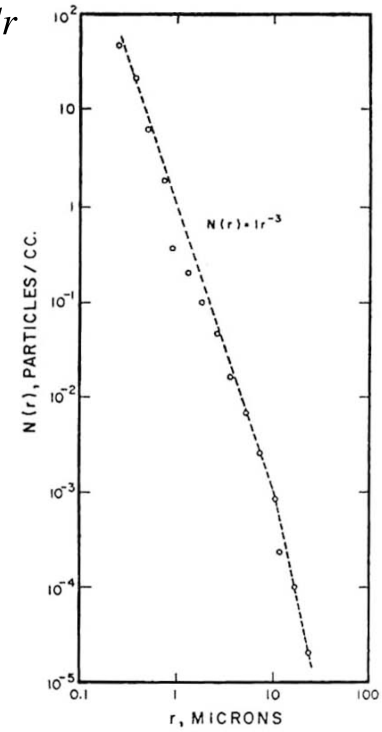
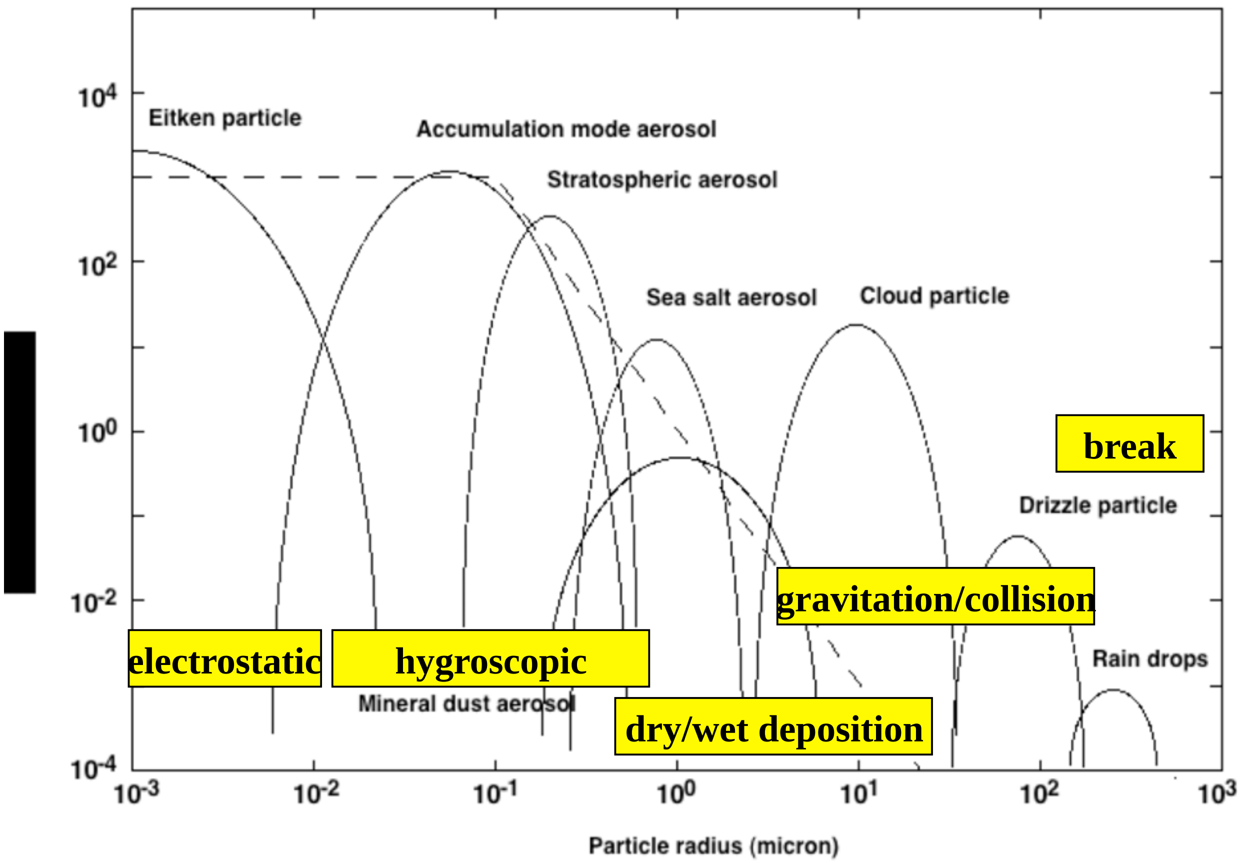


FIG. 3. Run 4 made in the laboratory on 22 February 1962. Over the radius range 0.3 to 10 microns the data are well represented by $N(r) \sim r^{-3}$.

Size distribution in the atmosphere



Ångtröm's law

- **Power law size distribution**
  p=4 : Junge

$$n(r)=\begin{cases} Cr^{-p} & if\ r>r_0 \\ Cr_0^{-p} & if\ r\le r_0 \end{cases}$$

- **Aerosol distribution tends to have a power law type size distribution, then**

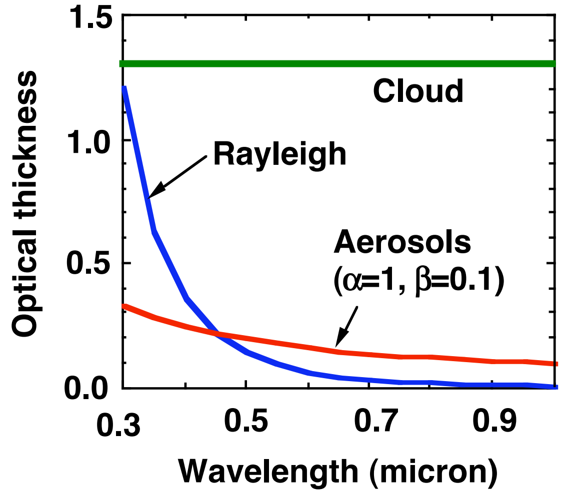
$$e=\int_0^\infty \pi r^2 Q(x) C r^{-p} dr = Ck^{p-3}\int_0^\infty \pi x^2 Q(x) x^{-p} dx$$

- **Ångström's law**

$$\tau=\beta\lambda^{-\alpha} \qquad \alpha=p-3$$

- **Junge distribution**

$$\alpha=1$$



Size distribution in the ocean (1)

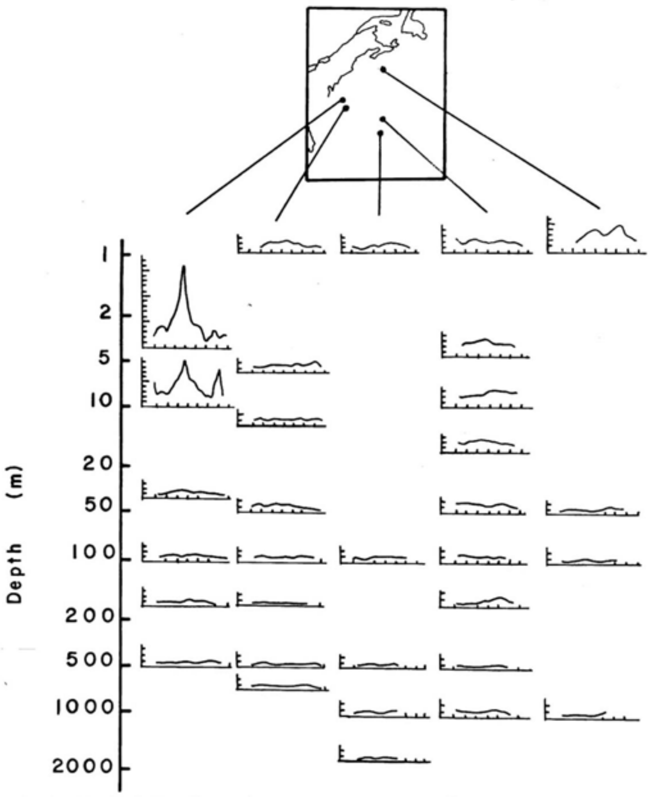


FIG. 8. Size distributions of suspended particulate matter at various depths in the western North Atlantic.

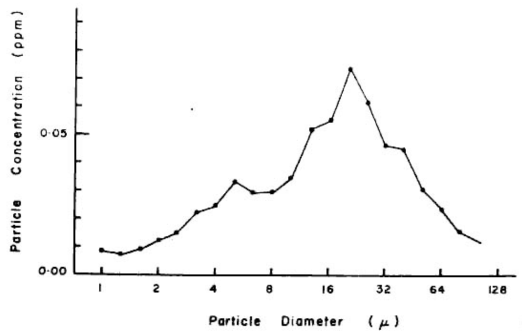
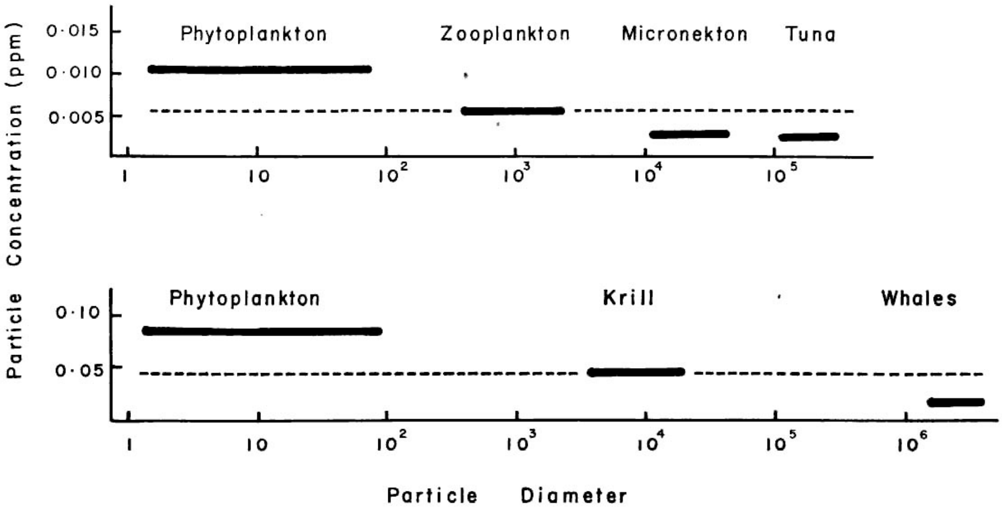


FIG. 2. Size-frequency distribution; to show the notation of the axes and the least number of data points used to define the form of the distribution. Concentration is *by volume*. All the size-frequency distributions in Figs. 3–9 were constructed in this way.

Sheldon et al. (1972)

Size distribution in the ocean (2)



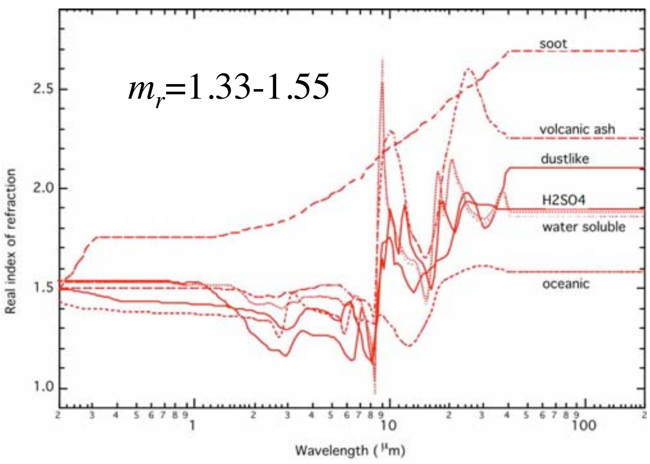
$$\frac{dV}{d\ln r} = \frac{4\pi r^3}{3}\frac{dN}{dr}\frac{dr}{d\ln r} = \frac{4\pi r^4}{3}n(r)$$

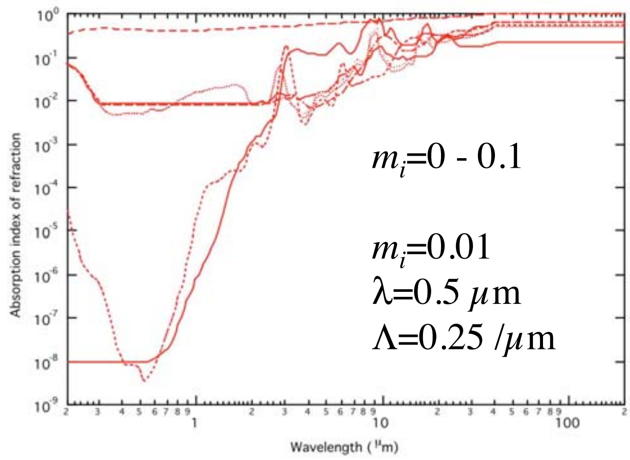
Sheldon et al. (1972)

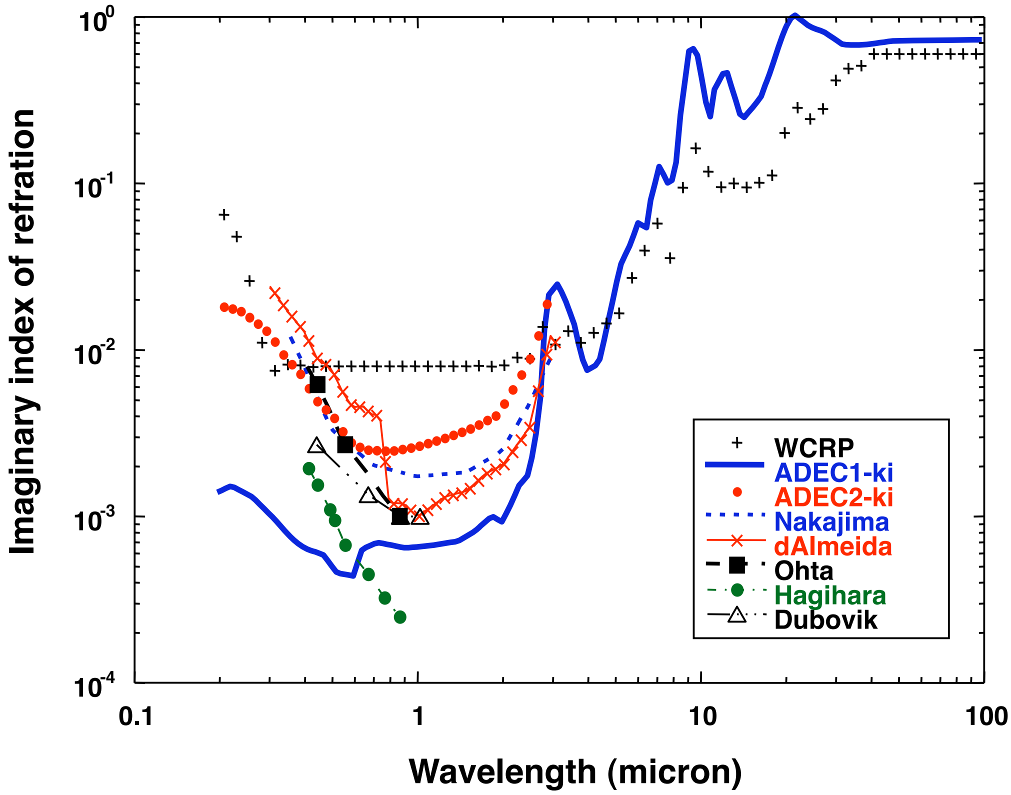
Refractive index of aerosols

- **Particle types**
  - **Water, ice**
  - **Soot, volcanic ash, dust like, H2SO4, water soluble, oceanic**

$$|E|^2 = \left|E_0 e^{i(\omega t - \tilde{m}k_0 x)}\right|^2 = |E_0|^2 e^{-\Lambda x}, \quad \Lambda = 2m_i k_0 = \frac{4\pi m_i}{\lambda}$$







Nakajima et al. (2007)

Einstein-Smolukowski theory (1908, 1910)

***Ma*=29 g/mole, *Mw*=18 g/mole**

***m*(air atm/m3)= 1000litre/22.4*29E-3=1.29kg**

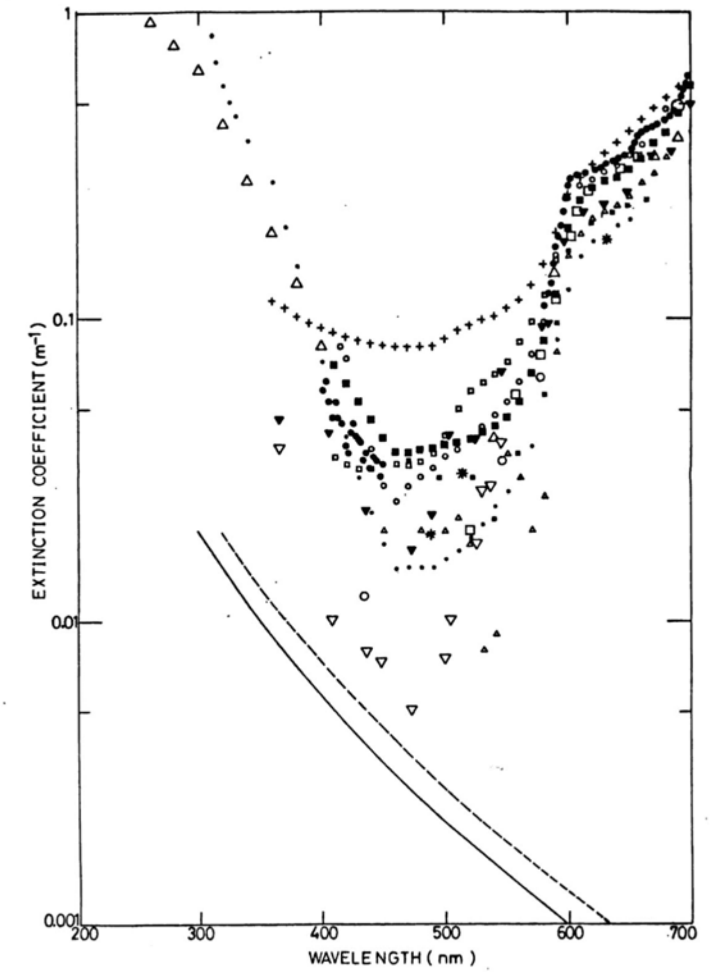
***m*(water/m3)= 1000kg**

***Nwater*/*Nair*=(*m*(water)/*Mw*)/(m(air)/*Ma*)=1250**

$$C_{s,water}N_{water} \approx 1250 C_{s,air}N_{air} = 0.0013\lambda_{micron}^{-4} \quad (1/m)$$???

Observed value for water:

$$C_{sca} = (1.934 + 0.9017\lambda^{-4.5}) \times 10^{-4} \quad (1/m)$$



Extinction and scattering coefficients of pure water